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On modeling the interaction of neoclassical and classical tearing modes with rf waves

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I. Introduction

A crucial issue for any long pulse, high temperature tokamak is the appearance of neoclassical tearing modes (NTMs). NTMs are slowly growing non-ideal MHD instabilities that produce magnetic islands at low order rational surfaces. The free-energy source for the instability is the bootstrap current which can produce islands when resistive MHD predicts stability ($\Delta' < 0$). Unlike many MHD instabilities, NTMs are metastable; they are linearly stable but nonlinearly excited at sufficient amplitude. When excited, NTMs can produce island widths that are substantial fractions of the minor radius (for β_θ values of interest to most tokamak experiments), cause significant reduction of energy confinement, and potentially lead to locked modes, loss of H-mode and/or disruption. Empirical observations indicate that the critical beta for neoclassical tearing mode onset scales with normalized ion gyroradius $\rho^*=\rho_i/a$, an extremely unfavorable scaling for most large tokamaks including ITER. Hence, methods to suppress the growth and appearance of NTMs are required. A prominent and highly successful method for NTM suppression is through the application of localized current drive in the magnetic island region. To date, the preferred tool of choice is electron cyclotron current drive (ECCD). Since various uncertainties in theoretically predicting the nonlinear island width threshold, seed island mechanisms, and the required RF suppression properties, NTM physics is one of the key MHD science questions to be addressed in a burning plasma experiment. In this document, we outline the beginnings of a program to address modeling issues of relevance to the coupled RF/MHD problem using a coupled theoretical/ computation approach.

The most commonly used paradigm for modeling NTMs is through the use of modified Rutherford equations. Such a treatment is valid if the magnetic island width exceeds the linear layer width. In this limit, the nonlinear $\boldsymbol{J} \times \boldsymbol{B}$ forces overwhelm the inertia and the vicinity of the rational surface can be treated as in nearly MHD equilibrium (relative to Alfven times) and slowly evolving on the resistive diffusion time through the island region. The quasineutrality equation ($\nabla \boldsymbol{J}=0$) in the vicinity of the island in the zero pressure gradient limit ($\boldsymbol{J}_\perp=0$) is given by

$$\vec{B}\cdot\nabla\frac{J_\parallel}{B}=0,$$

which has the solution

$$\vec{J}=f(\Psi^*)\vec{B},$$

where f is a function of the helical flux surface label Ψ^*. In order to find f, resistive Ohm's law can be used

$$< \vec{E} \cdot \vec{B} >_{*} = \eta < \vec{J} \cdot \vec{B} >_{*},$$

$$- < \frac{\partial \vec{A}}{\partial t} \cdot \vec{B} >_{*} = \eta < B^2 >_{*} f(\Psi^*),$$

where the bracket denotes an average over helical magnetic surfaces. The only surviving term from the average over the parallel electric field is due the induced electric field from the temporally growing magnetic island producing perturbation. If the island width is small relative to the minor radius, an asymptotic matching process, as used in the linear theory, can be employed. The conventional resistive-MHD prediction with zero pressure gradient leads to an island evolution equation of the form

$$\frac{dw}{dt} = \frac{\eta}{\mu_o} k_1 \Delta'$$

where w is the island width, η is the plasma resistivity, μ_o the permeability of free space, k_1 (~ 1.2) and Δ' is the asymptotic matching index. For unstable tearing modes ($\Delta' > 0$), the island width grows linearly in time and ultimately saturates due to quasilinear flattening of the current profile.

The electron viscous stress tensor modifies the Ohm's law used in fluid theory. Neoclassical theory accounts for the effect of inhomogeneous magnetic fields. In axisymmetric toroidal geometry, this leads to a damping of the electron fluid flow in the poloidal direction that ultimately produces a neoclassical modification to the Spitzer resistivity and a bootstrap current, a parallel current driven by cross field density and temperature gradients. The inclusion of neoclassical physics in Ohm's law leads to the possibility of pressure induced magnetic islands. The addition of the neoclassical modification to Ohm's law leads to a modified Rutherford equation of the form

$$\frac{dw}{dt} = \frac{\eta_{nc}}{\mu_o} k_1 [\Delta' + \Delta_{bs}(w)]$$

that produces a new term to the island evolution equation and accounts for the trapped particle correction to the plasma resistivity. In the large island limit, the new term scales as $\Delta_{bs} \sim D_{nc}/w$ with $D_{nc} \sim \varepsilon^{0.5} \beta_\theta L_q/L_p$. Essentially, this is a measure of the local bootstrap current on the helical magnetic surfaces outside the island separatrix with $L_q = q/q'$, $L_p = - p/p'$. The physics of this instability can be understood as a consequence of the self-consistent deformation of the bootstrap current profile. As the island grows, the pressure profile equilibrates along the helical field lines of the magnetic island. This leads to a helically resonant flatspot in the bootstrap current profile inside the island separatrix. This produces a magnetic resonant perturbation that is destabilizing in conventional tokamak operation with $q' > 0$. When $\Delta' < 0$, the saturated island width is given by

$$w_{sat} = \frac{D_{nc}}{(-\Delta')},$$

which can be an appreciable fraction of the minor radius.

For islands sizes smaller than the characteristic saturated value, a number of additional physical effects modify the island behavior. Diamagnetic currents, polarization currents and ion viscous forces produce contributions. Including these effects in the quasineutrality relation gives the relation

$$\vec{B}\cdot\nabla\frac{J_{\parallel}}{B} = -\nabla\cdot\left(\frac{\vec{B}\times\nabla p}{B^2} + \frac{\vec{B}\times\rho\frac{d\vec{v}}{dt}}{B^2} + \frac{\vec{B}\times\nabla\cdot\overleftrightarrow{\Pi}_i}{B^2}\right),$$

where the ion viscous stress includes contributions from both parallel and gyrovisosity. Inverting the ***B·∇*** operator on the lwft side yields additional parallel currents that affect island evolution. Including these effects yields a modified Rutherford equation of the form

$$\frac{dw}{dt} = \frac{\eta_{nc}}{\mu_o}k_1[\Delta' + \Delta_{bs}(w) + \Delta_{\text{int}}(w) + \Delta_{pol}(w)]$$

where the effects of resistive interchange and neoclassical polarization physics enter in through $\Delta_{int}(w)$ and $\Delta_{pol}(w)$, respectively. All of the terms on the right depend on the details of the model used; but are typically sub-dominant to the first two terms when the island is large. However, they give important corrections when the island width is small.

Of particular relevance to NTM physics is the effect of anisotropic heat flux, namely the competition between parallel and perpendicular diffusion. As mentioned previously, the NTM destabilization mechanism depends upon the self-consistent equilibration of pressure profiles along field lines. For sufficiently virulent cross-field transport (or sufficiently weak parallel transport), self-consistent flattening of the pressure profile in the island region does not occur and the neoclassical tearing instability is not active. Analysis of a temperature evolution equation with phenomenological cross-field ($\chi_\perp$) and parallel ($\chi_{||}$) heat diffusivities yields a characteristic island width ~ $(\chi_\perp/\chi_{||})^{0.25}$ below which does not allow effective pressure profile equilibration. This effect introduces a finite island width threshold. Magnetic islands that are smaller than this threshold value are not sufficiently destabilized by the bootstrap current for NTM physics to occur. Neoclassical polarization effects can also produce magnetic island thresholds, although a precise understanding of this effect is a topic of research. Crudely, the neoclassical polarization effects are important when the island width is comparable to the ion banana width. For islands below this characteristic value, the ion response is non-local and a more detailed kinetic theory needs to be worked out. Again, this is a topic of research.

In order for the NTM growth to be initiated, a magnetic island most be introduced at a level larger than the threshold value. This is referred to as the seeding problem. There are a number of theories for how seeding might occur; none of these are universally accepted as accounting for all of the experimental observations.

In an effort to combat the deleterious effects of the neoclassical tearing mode, a campaign to use localized current drive [principally electron cyclotron current drive (ECCD)] was initiated. The effect of an RF induced force on the electron fluid has principally two effects, the modification of the bulk current profile (hence effecting Δ') and the addition of a new term in the island region that alters the modified Rutherford equation.

$$\frac{dw}{dt} = \frac{\eta_{nc}}{\mu_o}k_1[\Delta' + \Delta_{bs}(w) + \ldots + \Delta_{RF}(w)],$$

The RF term depends upon the profile of the RF source in the island region. The optimal case corresponds to an RF induced source located inside the island separatrix that essentially replaces the "missing" bootstrap current. The analytic modeling that lead to the above equation used a modified Ohm's law of the form

$$\vec{E} + \vec{v} \times \vec{B} = \eta J + \ldots \vec{F}_{rf}$$

with $\boldsymbol{F}_{rf} = F_{rf}(\boldsymbol{x},t)\boldsymbol{B}$. The scalar F is a prescribed function of space. While qualitative details remain to be worked out, the simple analytic modeling has compared rather favorably with the growing experimental work in this area. ECCD stabilization of NTMs has been demonstrated on a number of tokamaks and is anticipated to be the tool of choice for stabilizing NTMs in ITER.

II. Elements of simulating NTM/RF modeling

We propose a multi-level approach for simulating the interaction of an RF source with magnetic islands in a toroidal plasmas. Crudely, there are three levels of sophistication that can be pursued somewhat in parallel.

- The first approach is a computational effort somewhat paralleling the simple analytic approach to model the interaction RF with magnetic island evolution by inserting an analytically chosen form for a source term in the Ohm's law.
- In the second approach, a phenomenological evolution equation will be used to describe the temporal and spatial structure of the source term.
- In the third approach, a more rigorous analytic problem will be solved where the inclusion of RF effects are treated as closure problems. The modified equations can then be implemented in numerical simulations.

In the second and third approaches, interfaces with the RF codes will be needed.

From the discussion of the effect of RF on magnetic islands, one will note that the RF term essentially enters the modified Rutherford equation as an additional term (as it enters in Ohm's law); the RF currents do not directly affect the neoclassical drive (at least to lowest order). Hence, information on the stabilizing properties of the RF terms can also be obtained from resistive MHD calculations without neoclassical or two-effects included.

It is important that computational efforts examining the long time scale behavior of tearing instabilities be re-initiated. Prior calculations of isolated resistive MHD and neoclassical tearing modes can be revisited with the newer versions of the fluid codes. Past modeling efforts in NIMROD for studying physics centered on using a "heuristic" model for the neoclassical electron viscous stress. Efforts to improve this closure scheme as well as efforts to include two-fluid, gyroviscous, etc. effects continue as part of the CEMM project. As these advancements materialize, more sophisticated fluid treatments of magnetic island physics can be used.

a. Phenomenological model for the RF current source

The simplest way to include the effects of RF current drive in a fluid code is to add an additional term to Ohm's law.

$$\vec{E} + \vec{v} \times \vec{B} = \eta J + \ldots \vec{F}_{rf} = \eta(\vec{J} - \vec{J}_{rf}) + \ldots$$

with $\boldsymbol{F}_{rf} = -\ \eta J_{rf}(\boldsymbol{x},t)\boldsymbol{B}/B$ with the scalar J_{rf} a function to be chosen. Such a calculation should allow the easiest and most direct comparison to the analytic theory described above. The crucial aspects of the current source are its amplitude, current channel width relative to the island width and the phase of the current source relative to the island phase.

b. Phenomenological evolution equation for the RF current source

At a higher level of sophistication, an evolution equation for the quantity J_{rf} can be used. One can view this approach is to include a simple RF "box" where information from the RF code enters. This approach basically models the work by Giruzzi and co-workers who used an additional evolution equation in the fluid evolution that accounts for a separate field quantity. Here, the physical effect of rapid parallel equilibration along the helical field lines is accounted for. While there are different versions of these types of models, one version is given in the form

$$\frac{\partial J_{RF}}{\partial t} + \chi_{\parallel}\nabla_{\parallel}^{2}J_{rf} + \chi_{\perp}^{2}\nabla_{\perp}^{2}J_{rf} + \nu_{rf}J_{rf} = S.$$

where the source term S is where RF codes deposit information. Due to its similarity with the temperature evolution equation, there is experience in solving equations of this form with highly disparate rates of cross-field to parallel diffusion. In steady state, rapid equilibration along field lines leads to an RF source that is distributed along the field line. Precise details for the exact nature of S and various coefficients need to be more properly defined.

An additional concern with approach is that since the above equation couples to other fluid variables, new normal modes can appear in the system. These modes could lead to unstable feedback and produce numerical instabilities. If this becomes an initial, modified or new computational approaches made need to be developed.

c. Closure scheme for modeling RF modifications to fluid equations

While the options described in the prior two sections allow for an "easy" introduction into the RF/island coupling problem, it is desirable to derive a more rigorous model for use in the simulation. The approach described here describes the problem as a closure issue. The fluid equations are evolved with the addition of extra RF sources coupled with a closure scheme modified by the RF physics.

To begin with, let's consider a kinetic equation in the form

$$\frac{df}{dt} = C(f) + Q(f),$$

where the left side is the usual kinetic operator in phase space, *C(f)* is the collision operator and *Q(f)* represents the contribution due to RF induced fields. For many applications of interest (such as ECCD), we can model *Q(f)* as a quasilinear diffusion operator of the form

$$Q(f) = \frac{\partial}{\partial \vec{v}} \cdot \vec{D} \cdot \frac{\partial f}{\partial \vec{v}}$$

where the diffusion tensor *D* is needed from RF codes. Taking moments of our kinetic equation, we are left with the usual fluid equations augmented by additional terms from the RF source

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \vec{v}_s) = 0,$$

$$m_s n_s (\frac{\partial \vec{v}_s}{\partial t} + \vec{v}_s \cdot \nabla \vec{v}_s) = n_s q_s (\vec{E} + \vec{v}_s \times \vec{B}) - \nabla p_s - \nabla \cdot \pi_s + \vec{R}_s + \vec{F}_s^{rf},$$

$$\frac{3}{2} n_s (\frac{\partial T_s}{\partial t} + \vec{v}_s \cdot \nabla T_s) + n_s T_s \nabla \cdot \vec{v}_s = -\nabla \cdot q_s - \vec{\pi}_s : \nabla \vec{v}_s + Q_s + S_s^{rf},$$

where conventional notation is used. The additional terms due to the RF are given by

$$\vec{F}_s^{rf} = \int d^3\vec{v}\ m_s \vec{v}_s Q(f_s),$$

$$S_s^{rf} = \int d^3\vec{v}\ \frac{1}{2} m_s v_s'^2 Q(f_s),$$

with an assumption that the RF produces no particles.

$$Q(f) = \frac{\partial}{\partial \vec{v}} \cdot \vec{J}_{rf} \Rightarrow \int d^3\vec{v} Q(f) = 0,$$

and the variable **v′** is used to denote the deviation of the phase space velocity from the fluid variable. It is important to point out that the additional RF terms appear simply as functions of three spatial dimensions and time.

At this point, the above fluid equations are exact. However, we do have to address the usual closure problem; calculations for the stress tensors and heat fluxes are needed. Since we are mostly interested in the RF modification to Ohm's law, it seems the closest analogy is with the Spitzer problem. This problem proceeds as a perturbation theory in the small quantity E/E_D where E_D is the Dreicer electric field. Since, we imagine the RF contribution is comparable to E, we assert the following balance neE ~ F^{rf}.

Since we are imagining the RF terms are in some sense small, we can assert that to lowest order the distribution is Maxwellian with small corrections. While this may be a poor assumption for some types of RF heated plasmas, for the case of electron cyclotron current drive, this is a good approximation. With this assertion, note that the RF contributions to the fluid equations can now be written

$$\vec{F}_s^{rf} = \int d^3\vec{v}\ m_s \vec{v}_s Q(f_s) = \int d^3\vec{v}\ m_s \vec{v}_s Q(f_{Ms})$$

$$S_s^{rf} = \int d^3\vec{v}\ \frac{1}{2} m_s v_s'^2 Q(f_s) = \int d^3\vec{v}\ \frac{1}{2} m_s v_s'^2 Q(f_{Ms})$$

to good approximation. With the identification of a proper quasilinear diffusion operator, $\boldsymbol{F}^{rf}$ and S^{rf} are now expressed as functions of low order fluid moments and RF physics.

It's important to note that with this approach, the only thing that is needed from the RF codes is the form for *D* as a function of the phase space variables. The procedure is the fluid code hands the state variables of interest to the RF code, the RF code subsequently determines the tensor *D* as a function of three spatial variables, speed, pitch angle and time and returns this information to the fluid code to determine $\boldsymbol{F}^{rf}$, S^{rf} and *Q(f)* for use in determining the closures and fluid evolution.

Using a Chapman-Enskog-like (CEL) approach, a kinetic equation for the distortion F away from the Maxwellian is derived which can subsequently solved to obtain **q** and **π.** We write

$$f = f_M + F = n(\vec{x},t)[\frac{m_s}{2\pi T(\vec{x},t)}]^{3/2} e^{-\frac{m_s v'^2}{2T(\vec{x},t)}} + F$$

where F has no density, momentum or temperature moments. Following the usual CEL procedure where the fluid equations are used to evaluate dF_M/dt, we have

$$\frac{dF}{dt} - C(f_M + F) = ... + Q(f_M) - \frac{\vec{v}'\cdot\vec{F}^{rf}}{nT} f_M - \frac{2}{3}\frac{S^{rf}}{nT}(\frac{mv'^2}{2T} - \frac{3}{2}) f_M,$$

where the species subscript is suppressed for simplicity. Since F is a small distortion, the collision operator on the right side can be linearized. The … bits on the right denote the "usual" CEL source terms due temperature and flow gradients that drive heat flows and viscous stresses. The important modification from the RF contribution enters as additional source terms on the right side.

To make further progress, one needs to solve the kinetic equation. Efficient and accurate solutions to equations of this form have been a topic of interest to the CEMM project (mostly through Eric Held's efforts). Solutions to the above equations should also be amendable to approaches under investigation.

As a particularly important limit, one could re-examine the equivalent Sptizer problem augmented by the RF contributions. For simplicity, let's only consider the solution along the magnetic field and further assume we are looking at time independent, homogeneous plasmas. The parallel Ohm's law (electron equation of motion) of interest is given by

$$0 \cong -neE_{\|} + R_{\|} + F_{\|}^{rf}.$$

where a number of terms/effects are dropped for simplicity. Note the closure problem for this case is due to the moment of the collision operator that has the form

$$R_{\|} = ne\eta_o J_{\|} + m_e n_e \nu_e \frac{3}{5n_e T_e} q_{\|e} + ...,$$

with $\eta_0 = ne^2/m_e\nu_e$ (not the Spitzer resistivity). Hence $q_{||}$ (and higher order moments) require a solution to the kinetic equation. One can solve the kinetic equation using the usual prescription of expanding F in terms of Laguerre polynomials, take moments and solve the resultant matrix equation. This will lead to solutions for the higher order moments in terms of J, E, F^{rf}, etc. For the conventional Spitzer problem, it is sufficient to only go to matrices of dimension ~ three to obtain good agreement. For the problem with the RF term, it is not clear if this is the case. Further work is required.

Clearly, there are further extensions to this Spitzer-like problem that need to be addressed. These include, calculations in a bumpy cylinder, calculations in toroidal equilibrium, time-dependent processes, multiple length scale expansions, etc. Nonetheless, the closure scheme outlined above can lead to important insights as to how the more exact problem can be addressed.

III. Summary

To make progress on the problem of RF induced currents affect magnetic island evolution in toroidal plasmas, a set of research approaches are outlined. Three approaches can be addressed in parallel. These are:

- Analytically prescribed additional term in Ohm's law to model the effect of localized ECCD current drive
- Introduce an additional evolution equation for the Ohm's law source term. Establish a RF source "box" where information from the RF code couples to the fluid evolution
- Carry out a more rigorous analytic calculation treating the additional RF terms in a closure problem.

These approaches rely on the necessity of reinvigorating the computation modeling efforts of resistive and neoclassical tearing modes with present day versions of the numerical tools.

For the RF community, the relevant action item is

- RF ray tracing codes need to be modified so that general three-dimensional spatial information can be obtained.

Further, interface efforts between the two codes require work as well as an assessment as to the numerical stability properties of the procedures to be used.