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Dynamic Transport Simulation code including Plasma Rotation and Radial Electric Field

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- TASK/TX: Dynamic transport simulation code
- Physics included in the TASK/TX code
- Numerical results
- Summary and task for the future

Transport Modelling

- **Hierarchy of transport phenomena in toroidal plasmas**:
  - **TASK/TR**: Diffusive Transport Equations:
    — Gradient-flux relation: Stationary solution of equations of motion
    — Conventional way of transport simulations
  - **TASK/TX**: **Dynamic Transport Equations**: $\Longleftarrow$ **Main topic of this talk**
    — Flux-averaged multi-fluid equations
    — Including inertia terms in equations of motion
    — Coupling with Maxwell's equations
    — Transient analysis including plasma rotations and E_r
  - **TASK/FP**: Kinetic Transport Equations:
    — Bounce-averaged Fokker-Plank equations
    — Modification of momentum distribution functions
    — Integrating heating, current drive and kinetic stability analysis codes

Motivation of the TASK/TX Code

- **Transport Simulation Including Core and SOL Plasmas**
  - **Role of separatrix**
    - — Closed magnetic surface $\Longleftrightarrow$ Open magnetic field line
    - — Difference of dominant transport process
- **Transient Behavior of Plasma Rotation**
  - **Radial electric field**: Radial force balance (Gauss's law = Poisson's equation)
  - **Poloidal rotation**: Equation of motion
  - **Toroidal rotation**: Equation of motion
  - Equation of motion rather than transport matrix
- **Analysis including Atomic Processes**

1D Dynamic Transport Code: TASK/TX

- **Dynamic Transport Equation** (TASK/TX) M. Honda and A. Fukuyama, submitted to JCP
  - **A set of flux-surface averaged equations**
  - **Two fluid equations for electrons and ions**
    — Continuity equations
    — Equations of motion (radial, poloidal and toroidal)
    — Energy transport equation
  - **Neoclassical transport**
    — Poloidal viscosity $\Longrightarrow$ Emerging all the neoclassical effects
    — NCLASS module, Hirshman and Sigmar
  - **Turbulent transport**
    — Intrinsic ambipolar diffusion through poloidal momentum transfer
    — Thermal diffusivity and perpendicular viscosity
  - **Maxwell's equations, Poisson's equation (Gauss's law)**
  - **Slowing-down equations for beam ion component**
  - **Diffusion equations for two-group neutrals**

Model Equations

- **Fluid equations** for electrons and ions $(s = \mathrm{e}, \mathrm{i})$:

$$\frac{\partial n_s}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r n_s u_{sr}) + S_s$$

$$\frac{\partial}{\partial t}(m_s n_s u_{sr}) = -\frac{1}{r}\frac{\partial}{\partial r}(r m_s n_s u_{sr}^2) + \frac{1}{r} m_s n_s u_{s\theta}^2 - \frac{\partial}{\partial r} n_s T_s + e_s n_s (E_r + u_{s\theta} B_\phi - u_{s\phi} B_\theta)$$

$$\frac{\partial}{\partial t}(m_s n_s u_{s\theta}) = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 m_s n_s u_{sr} u_{s\theta}) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3 n_s m_s \mu_s \frac{\partial}{\partial r}\frac{u_{s\theta}}{r}\right) + e_s n_s (E_\theta - u_{sr} B_\phi)$$

$$+ F_{s\theta}^{\mathrm{NC}} + F_{s\theta}^{\mathrm{C}} + F_{s\theta}^{\mathrm{W}} + F_{s\theta}^{\mathrm{L}} + F_{s\theta}^{\mathrm{IN}} + F_{s\theta}^{\mathrm{CX}}$$

$$\frac{\partial}{\partial t}\left(m_s n_s u_{s\phi}\right) = -\frac{1}{r}\frac{\partial}{\partial r}(r m_s n_s u_{sr} u_{s\phi}) + \frac{1}{r}\frac{\partial}{\partial r}\left(r n_s m_s \mu_s \frac{\partial}{\partial r} u_{s\phi}\right) + e_s n_s (E_\phi + u_{sr} B_\theta)$$

$$+ F_{s\phi}^{\mathrm{C}} + F_{s\phi}^{\mathrm{W}} + F_{s\phi}^{\mathrm{L}} + F_{s\phi}^{\mathrm{IN}} + F_{s\phi}^{\mathrm{CX}}$$

$$\frac{\partial}{\partial t}\frac{3}{2} n_s T_s = -\frac{1}{r}\frac{\partial}{\partial r} r \left(\frac{5}{2} u_{sr} n_s T_s - \frac{3}{2} n_s \chi_s \frac{\partial}{\partial r} T_{\mathrm{e}}\right) + e_s n_s (E_\theta u_{s\theta} + E_\phi u_{s\phi})$$

$$+ P_s^{\mathrm{C}} + P_s^{\mathrm{L}} + P_s^{\mathrm{R}} + P_s^{\mathrm{RF}}$$

- **Slowing-down equations for beam ion component**

$$\frac{\partial n_\mathrm{b}}{\partial t} = S_\mathrm{b}^B - S_\mathrm{b}^\mathrm{C}$$

$$\frac{\partial}{\partial t}(m_\mathrm{b} n_\mathrm{b} u_{\mathrm{b}\theta}) = e_\mathrm{b} n_\mathrm{b} E_\theta + F_{\mathrm{b}\theta}^\mathrm{C} + F_{\mathrm{b}\theta}^\mathrm{IN} + F_{\mathrm{b}\theta}^\mathrm{CX}$$

$$\frac{\partial}{\partial t}\left(m_\mathrm{b} n_\mathrm{b} u_{\mathrm{b}\phi}\right) = F_{\mathrm{b}\phi}^\mathrm{C} + F_{\mathrm{b}\phi}^\mathrm{IN} + F_{\mathrm{b}\phi}^\mathrm{CX} + F_{\mathrm{b}\phi}^\mathrm{B}$$

- **Diffusion equations for two-group neutrals (fast and thermal)**

$$\frac{\partial n_0}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left(-r D_0 \frac{\partial n_0}{\partial r}\right) + S_0$$

- **Maxwell's equations**

$$\frac{1}{r}\frac{\partial}{\partial r}(r E_r) = \frac{1}{\epsilon_0}\sum_s e_s n_s$$

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial E_\phi}{\partial r}, \qquad \frac{\partial B_\phi}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r E_\theta)$$

$$\frac{1}{c^2}\frac{\partial E_\theta}{\partial t} = -\frac{\partial}{\partial r} B_\phi - \mu_0 \sum_s e_s n_s u_{s\theta}, \qquad \frac{1}{c^2}\frac{\partial E_\phi}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(r B_\theta) - \mu_0 \sum_s e_s n_s u_{s\phi}$$

Transport Model

- **Neoclassical transport**
  - Parallel viscosity force due to a poloidal plasma rotation
  - Valid for all three neoclassical regimes

$$F_{s\theta}^{\mathrm{NC}} \equiv -n_s m_s \nu_{\mathrm{NC}s} u_{s\theta} = -\frac{\langle B^2 \rangle \hat{\mu}_{11}^{si}}{n_s m_s B_\theta^2} n_s m_s u_{s\theta}$$

$\hat{\mu}_{11}^{si}$: viscosity coefficient from the NCLASS module, W. A. Houlberg et al. PoP **4** (1997) 3230

- **Due to the poloidal viscosity force,**
  - Neoclassical diffusion and Ware pinch
  - Resistivity and bootstrap current
- **Turbulent diffusion**
  - Poloidal momentum exchange between electrons and ions through turbulent fluctuating field
  - Intrinsic ambipolar flux (electron particle flux = ion particle flux)

$$F_{\mathrm{e}\theta}^{\mathrm{W}} = -F_{\mathrm{i}\theta}^{\mathrm{W}} = -\frac{e^2 B_\phi^2 D_\mathrm{e}}{T_\mathrm{e}} n_\mathrm{e} \left(u_{\mathrm{e}\theta} - \frac{B_\theta}{B_\phi} u_{\mathrm{e}\phi} \right)$$

$$F_{\mathrm{e}\phi}^{\mathrm{W}} = -F_{\mathrm{i}\phi}^{\mathrm{W}} = \frac{e^2 B_\phi^2 D_\mathrm{e}}{T_\mathrm{e}} \frac{B_\theta}{B_\phi} n_\mathrm{e} \left(u_{\mathrm{e}\theta} - \frac{B_\theta}{B_\phi} u_{\mathrm{e}\phi} \right)$$

- **Perpendicular viscosity**
  - Non-ambipolar particle flux (electron particle flux ≠ ion particle flux)

Modelling of SOL Plasma

- **Parallel losses in the SOL**
  - **Particle, momentum and ion heat losses along the magnetic field: convection**
    — Decay time in a sound velocity time scale

$$\nu_L = \frac{k_L C_s}{2\pi q R} \quad (a < r < b)$$

  - **Electron heat loss: conduction**
    — Classical thermal diffusivity

$$\nu_L = k_L \frac{\chi_\parallel}{(2\pi q R)^2} = k_L \frac{\kappa_0 T_e^{5/2}}{n_e (2\pi q R)^2} \quad (a < r < b)$$

- **Particle source**

$$S_e = n_0 \langle \sigma_{ion} v \rangle n_e - \nu_L (n_e - n_{e,div})$$

- **Recycling from divertor**
  - Recycling ratio: $\gamma_0 = 0.8$
  - Fixed density and temperature at divertor
  - Neutral source:

$$S_0 = \frac{\gamma_0}{Z_i} \nu_L (n_e - n_{e,div}) - \frac{1}{Z_i} n_0 \langle \sigma_{ion} v \rangle n_e + \frac{P_b}{E_b}$$

- **Gas puff from wall**

Stationary Electron Flux

- **Stationary Electron Flux**
  - Setting inertia terms to zero in the model equations.
  - **Physics included in the model equations become clear.**
- **Radial velocity**

$$u_{er} = -\frac{1}{1+\alpha}\frac{\bar{\nu}_e + \nu_{eNC}}{n_e m_e \Omega_{e\phi}^2}\frac{\partial p}{\partial r} - \frac{\alpha}{1+\alpha}\frac{E_\phi}{B_\theta} + \frac{1}{1+\alpha}\frac{1}{n_e m_e \Omega_{e\phi}}\left(F_{e\theta}^{W} + \frac{B_\phi}{B_\theta}\alpha F_{e\phi}^{W}\right)$$

$$+ \frac{\alpha}{1+\alpha}\frac{1}{\Omega_{e\phi}}\frac{B_\phi}{B_\theta}\left[\nu_{eb}u_{b\phi} - (\bar{\nu}_e - \nu_{ei})u_{i\phi}\right] + \frac{1}{1+\alpha}\frac{\bar{\nu}_e + \nu_{eNC} - \nu_{ei}}{\Omega_{e\phi}}u_{i\theta}$$

where $\bar{\nu}_e \equiv \nu_{ei} + \nu_{eb} + \nu_L + \nu_{0e}$,

$$\alpha \equiv \frac{\bar{\nu}_e + \nu_{eNC}}{\bar{\nu}_e}\frac{B_\theta^2}{B_\phi^2}, \quad \Omega_{e\phi} \equiv \frac{eB_\phi}{m_e}, \quad \text{and} \quad \nu_{eb} \equiv \frac{n_b m_b}{n_e m_e}\nu_{be}$$

  - Damping rate, $\bar{\nu}_{eNC}$, due to the neoclassical viscosity
  - Factor α is a contribution of toroidicity.
  - First, second and third terms in RHS are **neoclassical diffusion**, **Ware pinch** and **turbulent diffusion**, respectively.
  - Fourth term denotes a neoclassical pinch due to a momentum input from beam ions.

- **Toroidal velocity**

$$u_{e\phi} = -\frac{1}{\bar{\nu}_e}\left[\frac{1}{1+\alpha}\frac{e}{m_e}E_\phi - \frac{1}{1+\alpha}\frac{B_\theta}{B_\phi}\frac{\bar{\nu}_e+\nu_{eNC}}{n_e m_e \Omega_{e\phi}}\frac{\partial p}{\partial r} + \frac{1}{1+\alpha}\frac{1}{n_e m_e}\left(\frac{B_\theta}{B_\phi}F^{W}_{e\theta} - F^{W}_{e\phi}\right) - \frac{\nu_{eb}}{1+\alpha}u_{b\phi}\right.$$
$$\left. + \frac{1}{1+\alpha}\frac{B_\theta}{B_\phi}(\bar{\nu}_e+\nu_{eNC}-\nu_{ei})u_{i\theta} - \frac{\nu_{ei}+\alpha\bar{\nu}_e}{1+\alpha}u_{i\phi}\right],$$

  - First, second and third terms in RHS are **neoclassical resistivity**, **bootstrap current**, and **turbulent driven current**.

- **Poloidal velocity can be obtained in a similar way.**

Model equations include major neoclassical effects!

Radial Mesh

- **Mesh generating algorithm**
  - The phenomenon near separatrix $(r = a)$ is important.
    $\Rightarrow$ **Necessary is the algorithm which allows us to obtain fine mesh around separatrix.**

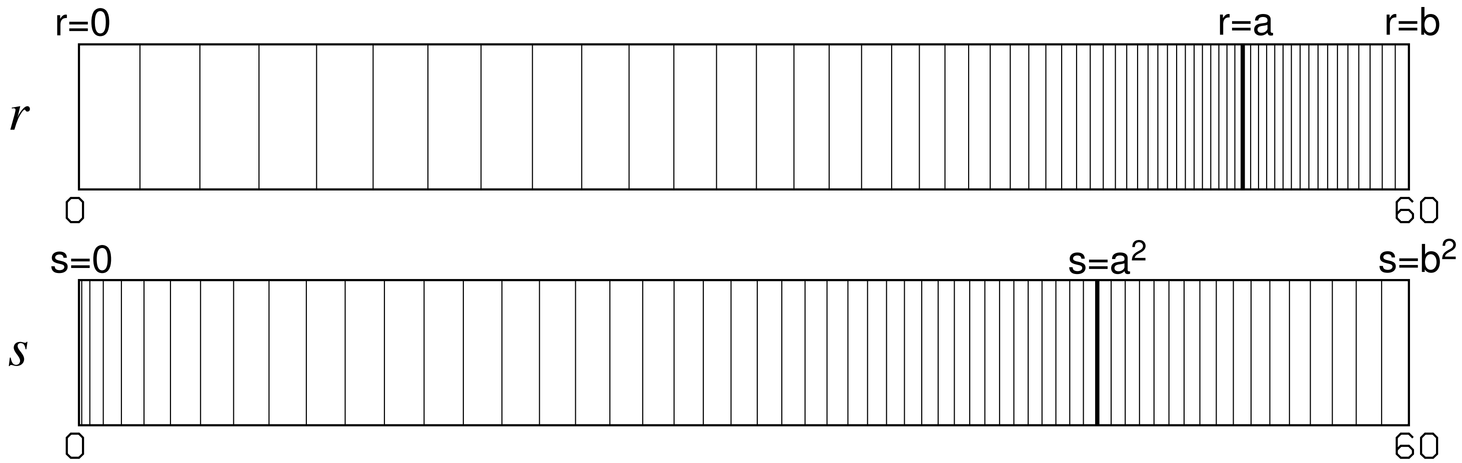
$$S = \int_0^r \frac{dw}{dr'} \mathrm{d}r', \quad \frac{dw}{dr} = C_0 \left(1 + \frac{C_1}{1 + [(r - r_s)/w]^2} \right)$$

C_0: normalization constantC_1: amplitude of peak r_s: accumulation point

w: width of accumulation region

Calculate inverse function of $r = r(S)$, where the function S is the one which is obtained from integrating the above Lorentzian function.

In the case of $C_1 \approx 8.016,\ r_s = a,\ w = 0.05$



Numerical Schemes Used in TASK/TX Code

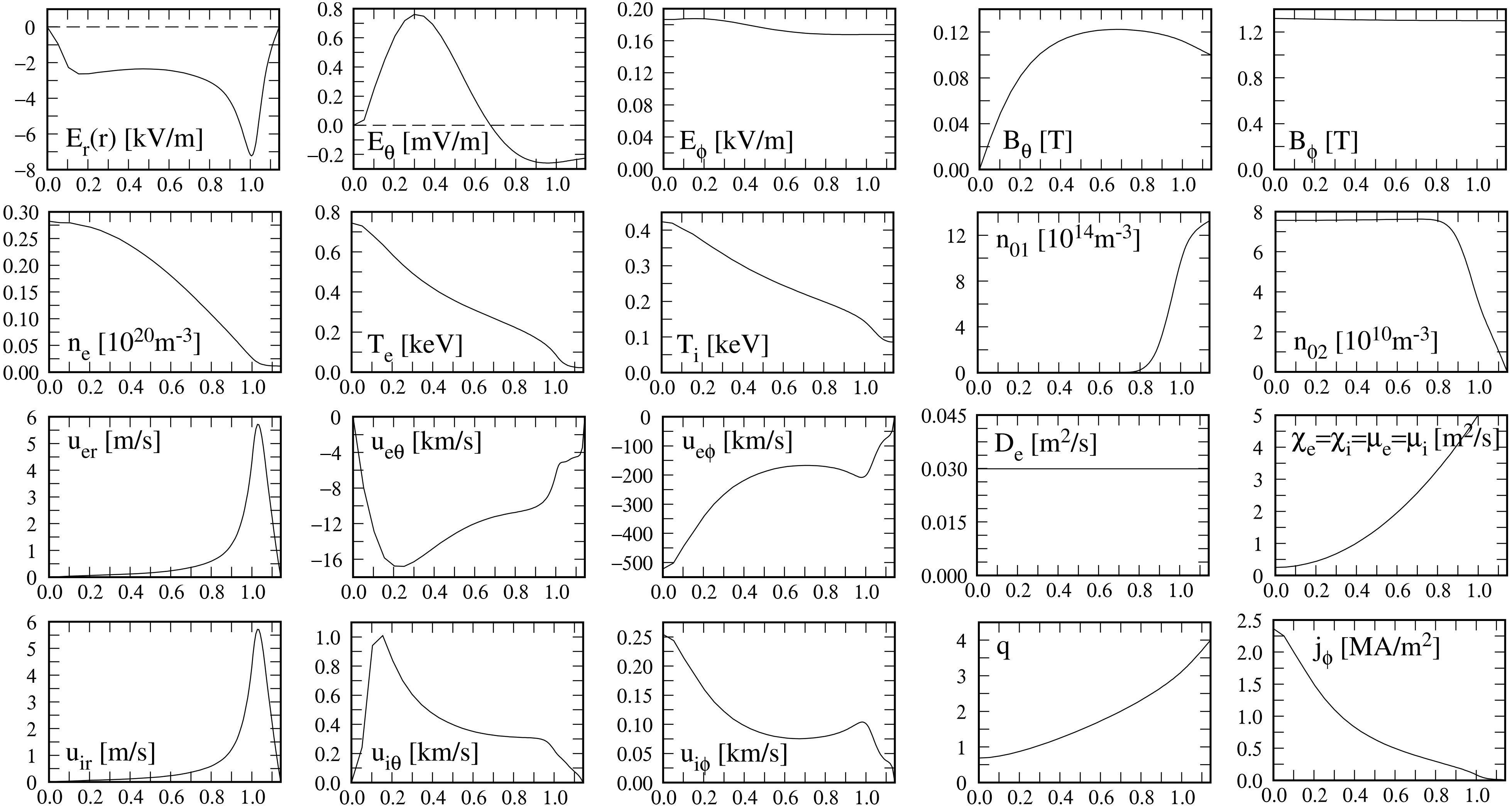
- **Finite element method (FEM)**
  - Linear interpolation function
  - **Streamline Upwind Petrov-Galerkin (SUPG) method**
    — Stabilizing spurious oscillation due to first-derivative terms
  - Transforming r coordinate to $s = r^2$ coordinate
    — Easy to impose natural boundary conditions at the magnetic axis
  - Careful choice of dependent variables to meet boundary conditions based on the characteristic of the linear interpolation function
  - **Achieving higher mesh resolution near the separatrix**
- **Time-advancing method**
  - **Full-implicit method**
    — Highly robust calculation
    — Time-consuming due to need of matrix equation solver
  - Mass lumping method
    — Avoidance of spurious diffusion due to use of consistent matrix in time-derivative terms
  - Picard method as solution algorithm for nonlinear equations
  - **LAPACK_DGBSV** for matrix equation solver
    — LU decomposition algorithm for band matrix calculation
    — High reliability, high efficiency

Typical Ohmic Plasma Profiles at $t = 50\,\mathrm{ms}$

- **JFT-2M like plasma** composed of electron and hydrogen

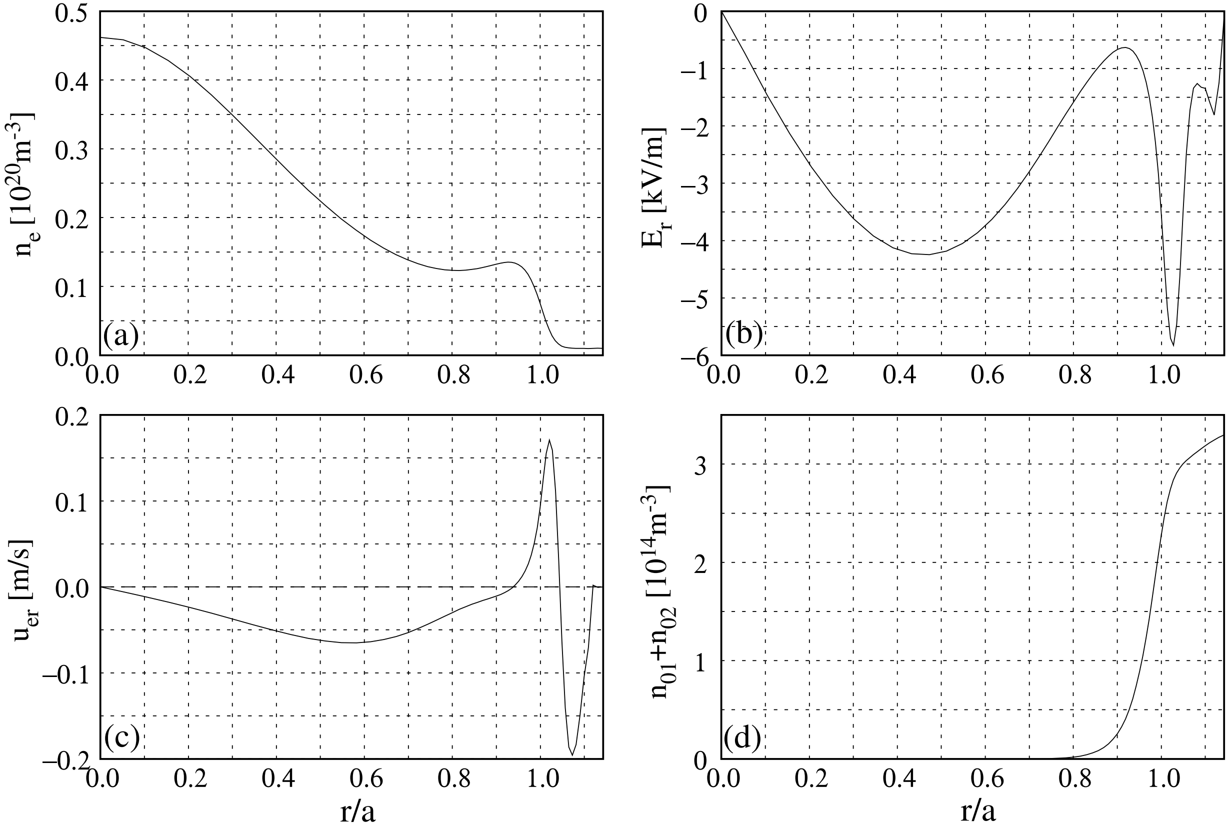
$$R = 1.3\,\mathrm{m},\ a = 0.35\,\mathrm{m},\ b = 0.4\,\mathrm{m},\ B_{\phi b} = 1.3\,\mathrm{T},\ I_p = 0.2\,\mathrm{MA},\ S_{\mathrm{puff}} = 5.0 \times 10^{18}\,\mathrm{m^{-2}s^{-1}}$$

$\gamma = 0.8$, $Z_{\mathrm{eff}} = 2.0$, Fixed turbulent coefficient profile



Neoclassical Transport without Turbulence at $t = 250\,\mathrm{ms}$

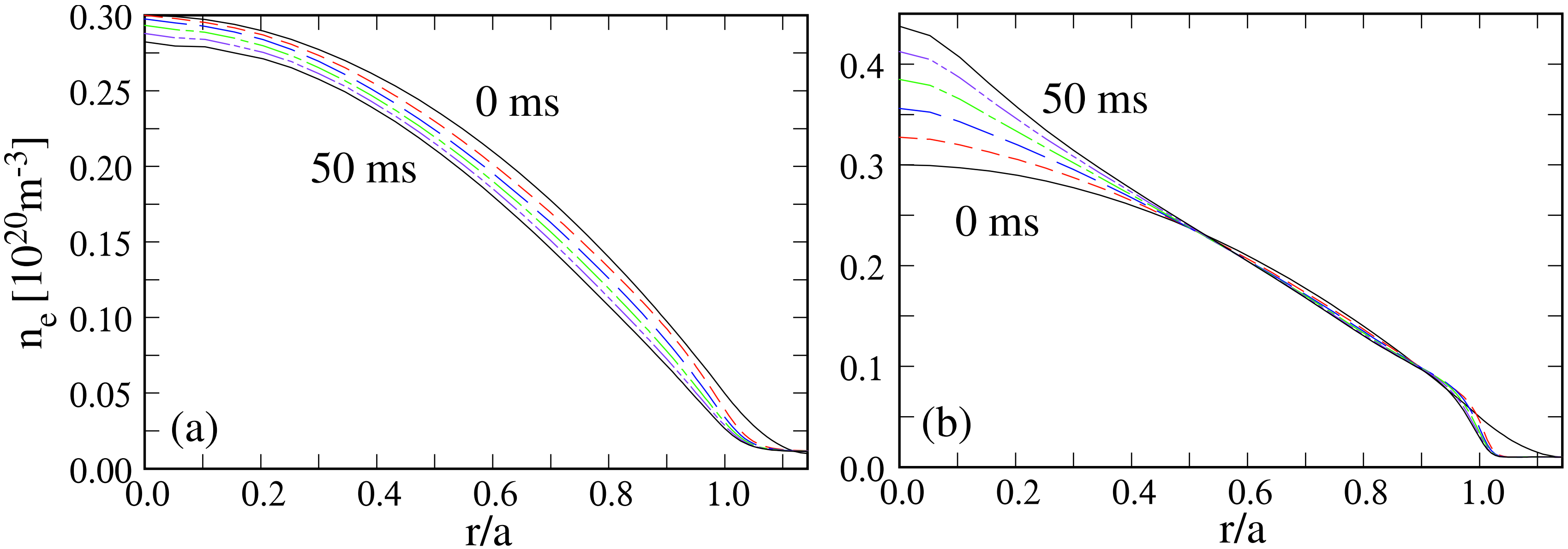
- **Clarifying neoclassical transport**
  - **No turbulent diffusivity and viscosity**
  - **Fixed temperature profiles**
  - Same parameters as those of typical profiles except $I_p = 0.12\,\mathrm{MA}$ and $S_{gas} = 3.0\times10^{18}\,\mathrm{m^{-2}s^{-1}}$
- **Density peaking with steep gradient near the separatrix at quasi-steady state**
  - **Density peaking due to Ware pinch from the neoclassical viscosity**
  - **Inward flux in the SOL due to ionization of neutrals**



Diffusion due to Turbulent Induced Force

- **Confirming the validity of our particle diffusion model**
  - **No particle diffusion terms appeared in the continuity equations**

    Turbulent induced poloidal friction force $\Longrightarrow$ Damping of poloidal velocity $\Longrightarrow$ Change of radial velocity $\Longrightarrow$ Particle diffusion through convective term
  - No neoclassical viscosity assumed in this case
- **Obvious particle diffusion observed in fig. (a)**
  - **No diffusion but only pinch observed if no turbulence imposed in fig. (b) due to turbulent viscosity**
- **Particle diffusion works properly in our model.**

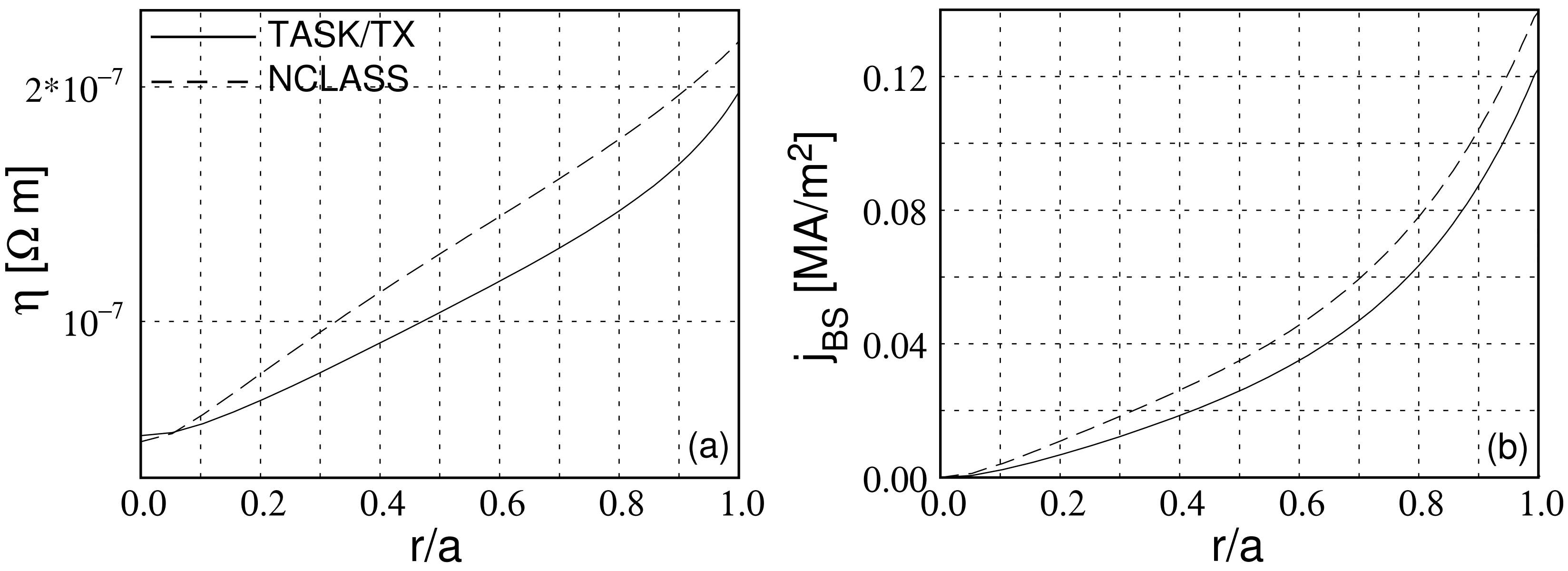


Validity of Neoclassical Transport Through Viscosity

- **Resistivity and bootstrap current derived from stationary flux**
  - Both are not explicitly calculated in the TASK/TX code.
  - **Analytic solutions obtained in the steady state**

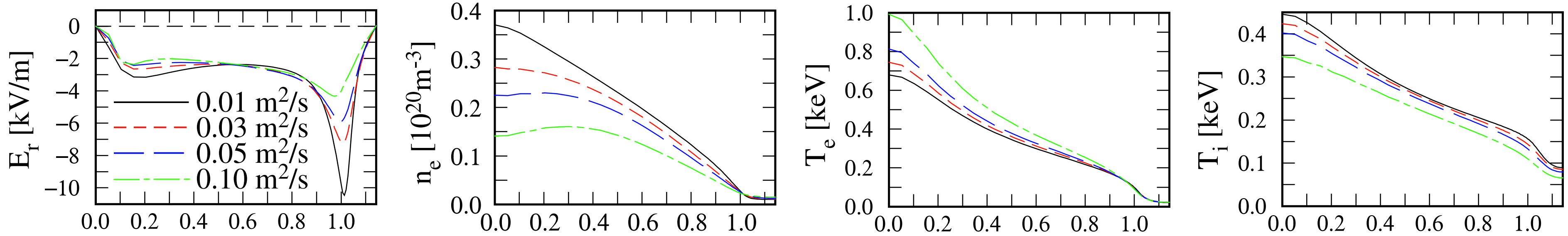
$$\eta = \frac{m_e(1+\alpha)\bar{\nu}_e}{n_e e^2}, \quad j_{BS} = -\frac{\alpha}{1+\alpha}\frac{1}{B_\theta}\frac{dp}{dr}$$

  - Assuming **flat temperature profile** in the core to minimize effect of neoclassical heat flux
  - **Comparing analytic solutions with direct outputs from NCLASS in the initial state**
- **Fairly good agreement**
  - Similar results can be obtained in spite of different ways to implement neoclassical effects.

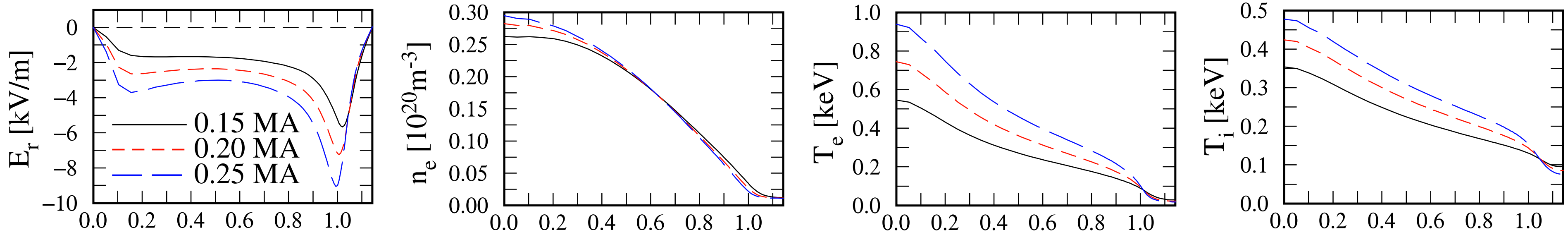


Parameter Dependence on D_e and I_p

- **Profile modifications due to the change of particle diffusivity**
  - **The cases of** $D_e = 0.01, 0.03, 0.05, 0.10\ \mathrm{m^2/s}$
  - **Density flattening with the increase of particle diffusion,** D_e
  - **Notch of** E_r **near the separatrix vanishes with the increase of** D_e **because of alleviation of** $\frac{\partial p}{\partial r}$.

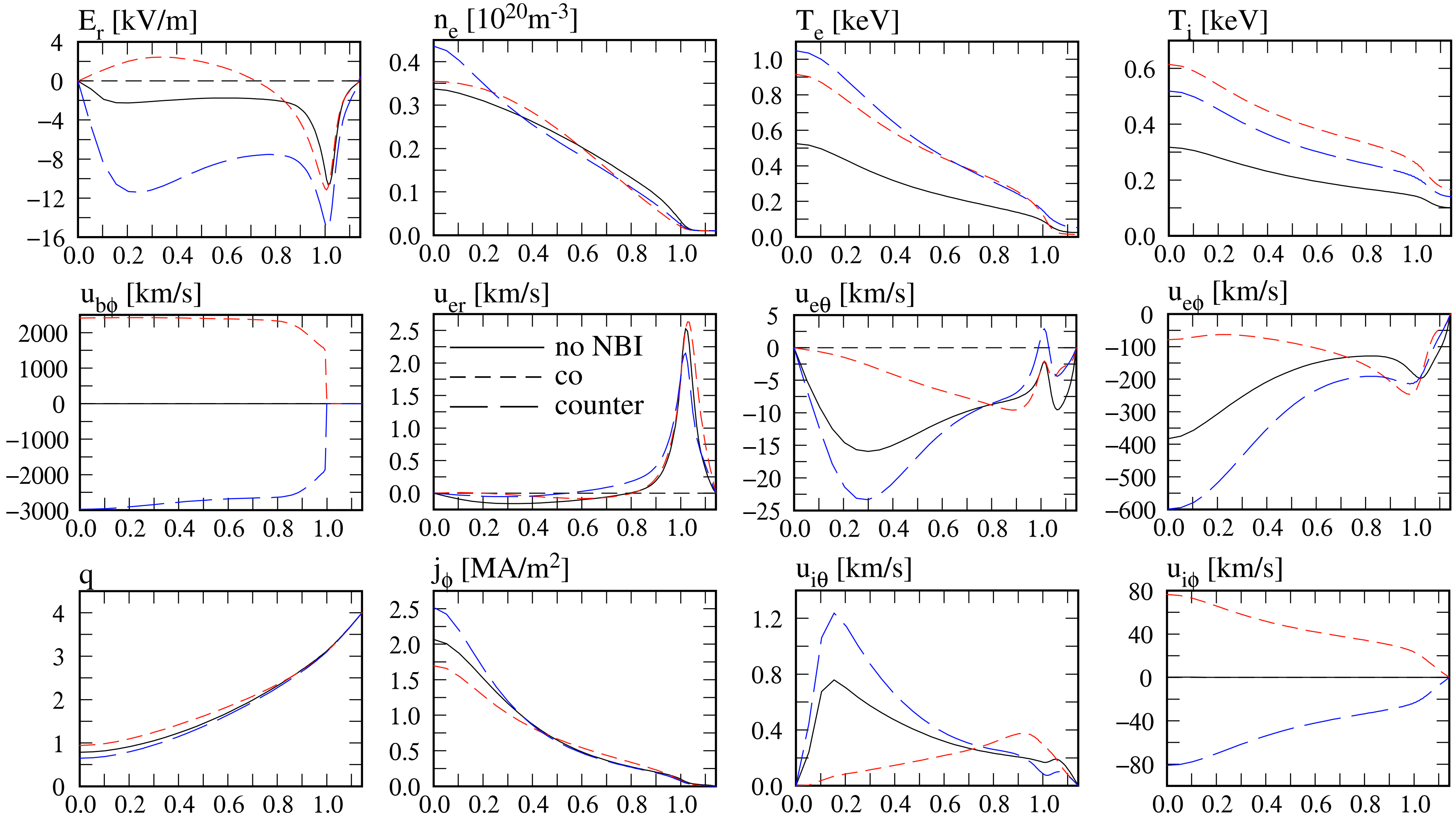


- **Profile modifications due to the change of plasma current**
  - **The cases of** $I_p = 0.15, 0.20, 0.25\ \mathrm{MA}$
  - **Increase in** n **and** T **near the axis and decrease in** E_r **with the increase of** I_p



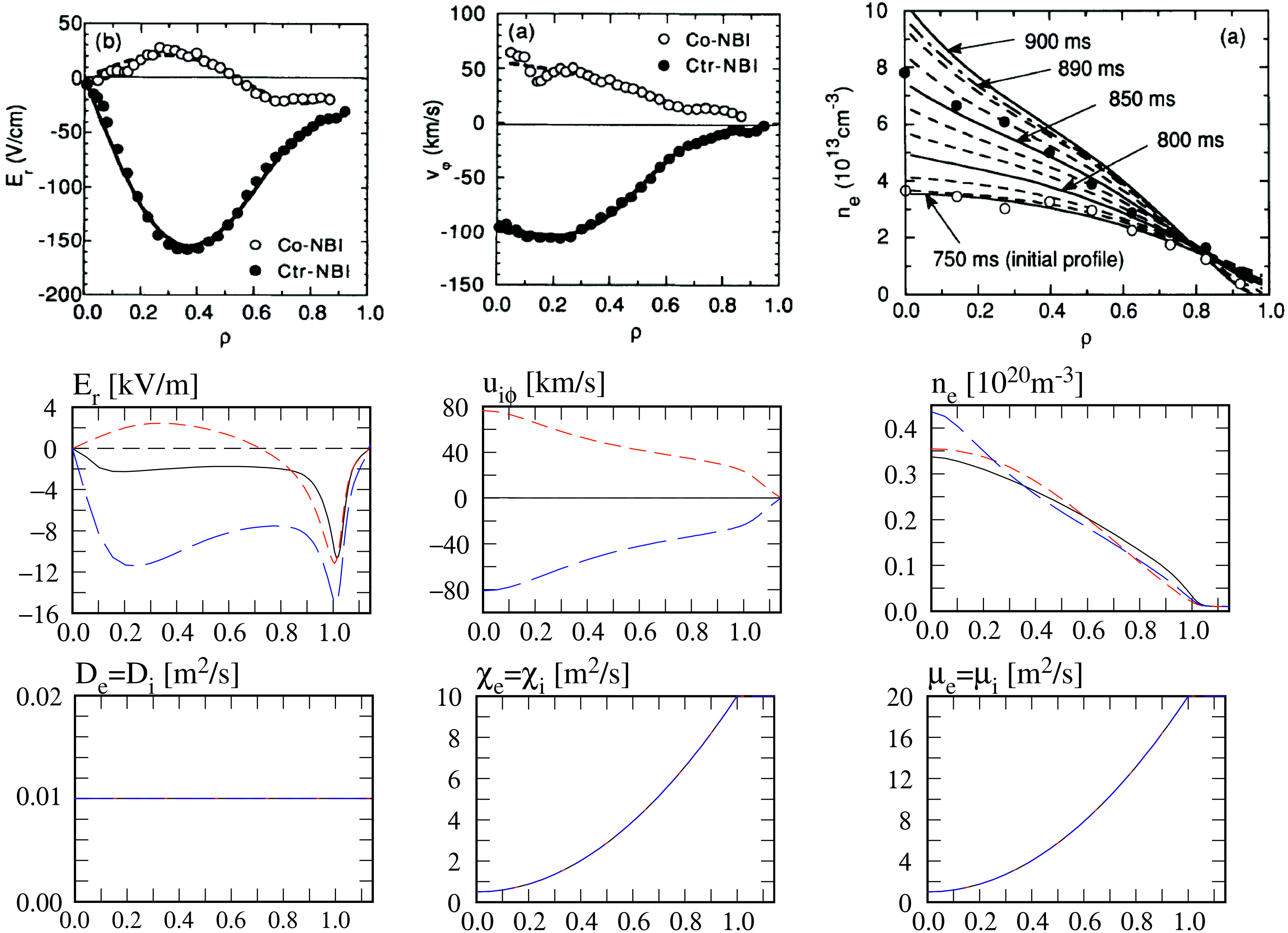
NBI of P_{NB} = 0.5 MW at t = 100 ms

- **The cases of before-, co- and ctr-NBIs**
  - **Modification of E_r profile depending on the direction of NBI, viz. $u_{i\phi}$**
  - **Co: $u_{b\phi} \nearrow \Rightarrow u_{i\phi} \nearrow \Rightarrow E_r \nearrow$, $u_{b\phi} \nearrow \Rightarrow u_{i\phi} \nearrow \Rightarrow u_{e\theta} \nearrow$ & $u_{e\phi} \nearrow \Rightarrow u_{er} \Rightarrow$ density flattening**
  - **Ctr: $u_{b\phi} \searrow \Rightarrow u_{i\phi} \searrow \Rightarrow E_r \searrow$, $u_{b\phi} \searrow \Rightarrow u_{i\phi} \searrow \Rightarrow u_{e\theta} \searrow$ & $u_{e\phi} \searrow \Rightarrow u_{er} \Rightarrow$ density peaking**



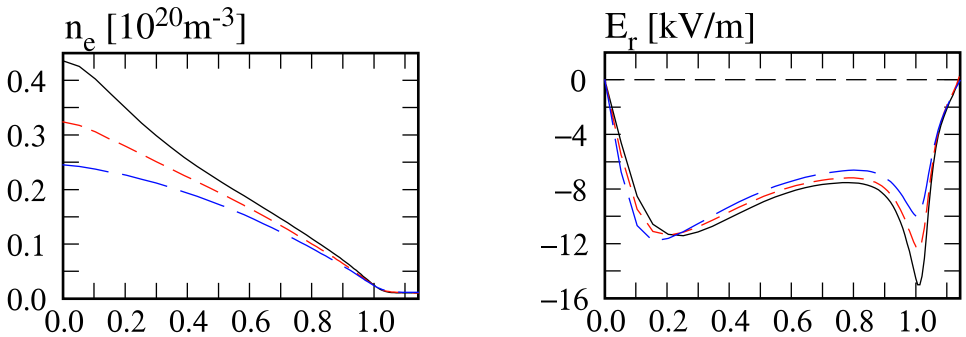
Comparison with JFT-2M experiment

Ref. K. Ida et al., PRL **68** (1992) 182



D_e dependence in the case of counter NBI

- **Density profile is determined by the balance between turbulent-driven and neoclassical particle fluxes during NBI.**
  - Density peaking vanishes when turbulent-induced force becomes large since particle diffusion exceeds particle inward pinch.
  - Density peaking appears when neoclassical effect is dominant because of the Ware pinch.
- **Ramp up particle diffusivity from 0.01 to 0.03 during counter NBI**
  - $D_e = 0.01$**: Density peaking**
  - $D_e = 0.02$: Almost the same profile before NBI
  - $D_e = 0.03$**: No peaking because of strong particle diffusion**
  - **Radial electric field is not significantly affected except near the separatrix.**



Black: $D_e = 0.01$, Red: $D_e = 0.02$, Blue: $D_e = 0.03$

Summary and Remaining Issues

- **TASK/TX**: We are developing the TASK/TX code in order to describe poloidal and toroidal rotations and radial electric field formation. The code simultaneously solves the two-fluid equations of motion; continuity equations and heat transport equations coupled with Maxwell's equations.

- **Numerical schemes**: For fine spatial resolution near the separatrix, FEM with linear interpolation function is introduced. Applying SUPG method and transforming radial coordinate of model equations has strongly improved the numerical stability of the code.

- **Neoclassical transport**: The NCLASS module is adopted for the evaluation of neoclassical viscosity. The validity of our approach of neoclassical transport is confirmed by comparing with the NCLASS results.

- **NBI**: We have analyzed the modifications of density profiles during NBI in JFT-2M like plasma. Density flattening in the case of co-NBI and density peaking in the case of ctr-NBI are qualitatively reproduced.

- **To-do list**
  - Including neoclassical heat flux
  - Transport simulation with theory-based turbulent transport model
  - Including impurity transport
  - Long time simulation with more numerical robustness