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CLOSED FLUID AND DRIFT-KINETIC ELECTRON FORMULATION FOR SLOW-MHD*

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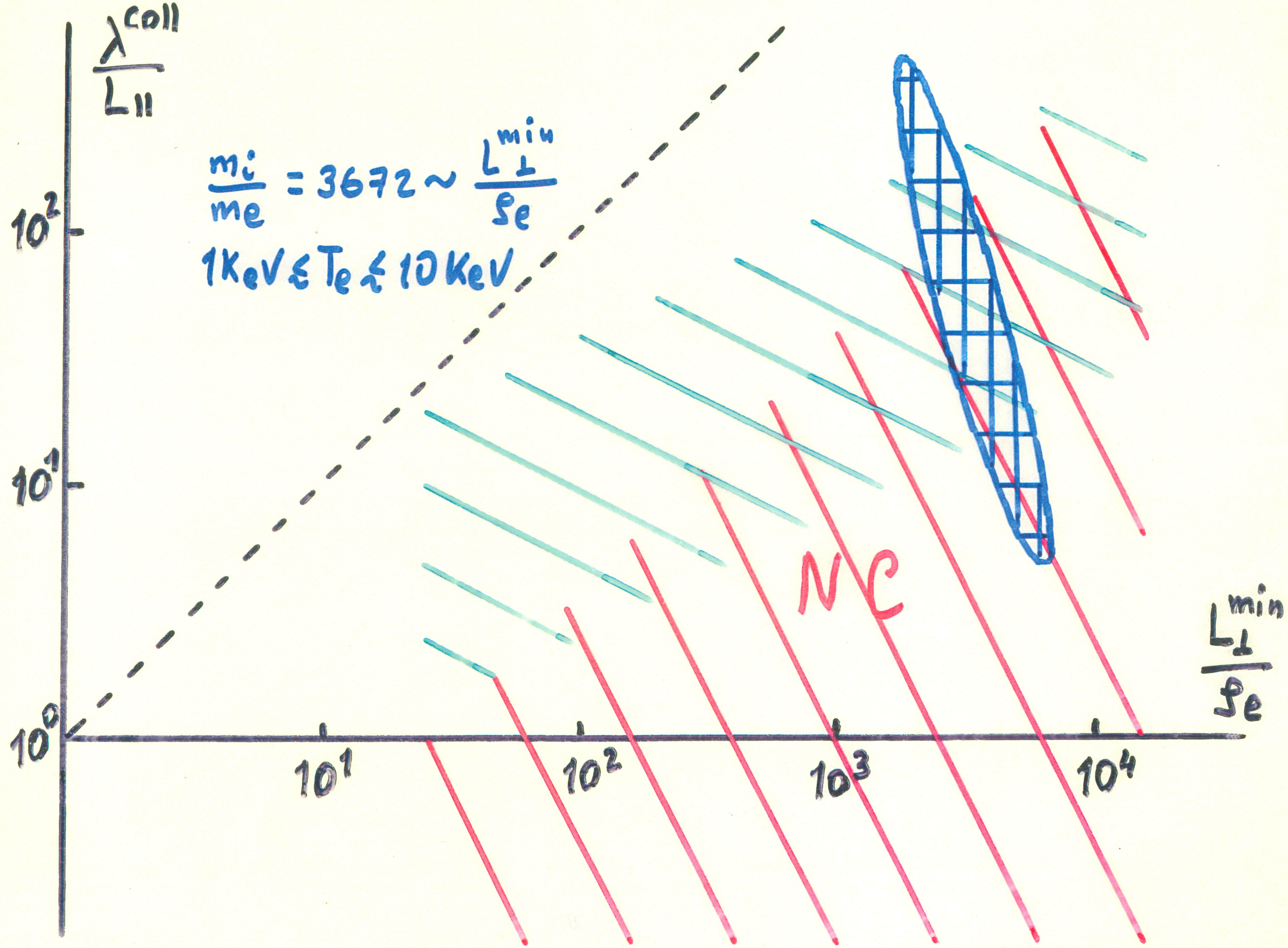
INTRODUCTION

A complete theoretical model of the electron dynamics for slow macroscopic plasma processes (in particular the "neoclassical" tearing instabilities interacting with applied ECCD) will be presented.

The model is a hybrid one, with fluid conservation equations for particle number, momentum and energy, and drift-kinetic closures.

Key to this work is a careful choice of the orderings relating fundamental parameters, aimed at describing as realistically as possible the low-collisionality, fusion-relevant plasmas of interest. The conventional ordering of the collisionality in neoclassical theory is deemed too high for the ions, even in the banana regime. Instead, the orderings $\rho_\iota/L \sim L/\lambda^{coll} \sim (m_e/m_\iota)^{1/2} \ll 1$ are adopted, which still yield a theory equivalent to the one based on the neoclassical banana orderings for the electrons.


$\frac{\lambda^{coll}}{L_{\parallel}}$
$\frac{m_i}{m_e} = 3672 \sim \frac{L_\perp^{min}}{\rho_e}$
$1 KeV \leq T_e \leq 10 KeV$
10^2
10^1
10^0
10^1
10^2
10^3
10^4
NC
$\frac{L_\perp^{min}}{\rho_e}$

BASIC FRAMEWORK AND ORDERING ASSUMPTIONS

Quasineutral plasma with one ion species of unit charge:

$$n_\imath = n_e = n \ , \qquad \mathbf{u}_e \ = \ \mathbf{u}_\imath \ - \ \frac{1}{en}\mathbf{j} \ ,$$

$$\frac{\partial n}{\partial t} \ + \ \nabla \cdot (n\mathbf{u}_\imath) \ = \ \frac{\partial n}{\partial t} \ + \ \nabla \cdot (n\mathbf{u}_e) \ = \ 0 \ ,$$

$$\mathbf{j} \ = \ \nabla \times \mathbf{B} \ , \qquad \frac{\partial \mathbf{B}}{\partial t} \ = \ - \ \nabla \times \mathbf{E} \ .$$

Electron kinetic equation:

$$\frac{\partial f_e}{\partial t} \ + \ \mathbf{v} \cdot \frac{\partial f_e}{\partial \mathbf{x}} \ - \ \frac{e}{m_e}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_e}{\partial \mathbf{v}} \ = \ C_{ee}(f_e, f_e) + C_{e\imath}(f_e, f_\imath) + S^{RF}(f_e),$$

where C_{ee} and $C_{e\imath}$ are Fokker-Plank collision operators and S^{RF} is an external RF source of momentum and energy: $\int d^3\mathbf{v} \ S^{RF}(f_e) = 0$ **.**

Small ion Larmor radius fundamental expansion parameter:

$$\delta \sim \rho_\iota / L \sim k\rho_\iota \ll 1 \ .$$

Small mass ratio and low collisionality orderings, linked to δ:

$$(m_e/m_\iota)^{1/2} \sim \delta \ , \qquad \textbf{hence} \qquad \rho_e/L \sim k\rho_e \sim \delta^2$$

and

$$\nu_\iota \sim \delta \nu_e \sim \delta^2 \Omega_{c\iota} \ , \qquad \textbf{hence} \qquad \lambda^{coll} \sim v_{th\iota}/\nu_\iota \sim v_{the}/\nu_e \sim \delta^{-1} L \ .$$

Macroscopic flows of the order of the diamagnetic drifts:

$$u_\iota \sim u_e \sim u_{*\iota,e} \sim \delta v_{th\iota} \sim \delta^2 v_{the} \ .$$

No further expansions based on geometrical or beta orderings:

$$L_\perp / L_\parallel \sim k_\parallel / k_\perp \sim \beta \sim 1 \ .$$

Close to Maxwellian distribution functions with comparable ion and electron temperatures and small parallel temperature gradients:

$$f_s = f_{Ms} + f_{NMs} = \frac{n}{(2\pi)^{3/2}\ v_{ths}^3} \exp\left(-\frac{|\mathbf{v} - \mathbf{u}_s|^2}{2\ v_{ths}^2}\right) + f_{NMs} \qquad \textbf{with} \qquad v_{ths}^2 \equiv T_s/m_s\ ,$$

$$\mathbf{b}\cdot\nabla T_s \sim \delta^2\ T_s/L, \qquad T_e \sim T_\iota, \qquad f_{NM\iota} \sim \delta\ f_{M\iota}, \qquad f_{NMe} \sim \delta^2\ f_{Me}\ .$$

These ordering assumptions are compatible with a consistent solution of the kinetic equations.

The electron collision operators are $C_{ee}(f_e, f_e) \sim C_{e\iota}(f_e, f_\iota) \sim \delta^2 \nu_e f_{Me} \sim \delta^3 (v_{the}/L) f_{Me}$**.**

The RF source is assumed to be comparable to the collisions: $S^{RF}(f_e) \sim \delta^3 (v_{the}/L) f_{Me}$**.**

Using $\Omega_{c\iota}$ **as reference, we have the following hierarchy of time scales:**

$$O(\delta^{-2}):\ \ \Omega_{ce} = v_{the}/\rho_e$$

$$O(1):\quad \Omega_{c\iota} = v_{th\iota}/\rho_\iota \sim v_{the}/L$$

$$O(\delta):\quad \nu_e \sim \omega_A = kc_A \sim \omega_S = kc_S \sim v_{th\iota}/L$$

$$O(\delta^2):\quad \nu_\iota \sim ku_{\iota,e} \sim \omega_{*\iota,e} = ku_{*\iota,e}$$

$$O(\delta^3):\quad collisional\ \ transport$$

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FLUID AND DRIFT-KINETIC APPROACH

Non-Maxwellian parts of the distribution functions, f_{NMs}, evaluated in the moving reference frames of their macroscopic flows, like the Maxwellian parts.

1**,** $\mathbf{v}-\mathbf{u}_s$ **and** $|\mathbf{v}-\mathbf{u}_s|^2$ **velocity moments of** f_{NMs} **equal to zero.**

Density, flow velocities and temperatures determined by fluid moment equations.

Solution of drift-kinetic equations for f_{NMs} to provide the fluid closure terms. Since f_{NMs} are obtained in the reference frames of their macroscopic flows, the evaluation of the stress and heat flux tensors is direct without the need of substracting the mean flows.

ELECTRON FLUID MOMENTUM EQUATION

Keeping $O(nm_e v_{the}^2/L) + O(\delta n m_e v_{the}^2/L) + O(\delta^2 n m_e v_{the}^2/L) + O(\delta^3 n m_e v_{the}^2/L)$**:**

$$m_e n \frac{\partial \mathbf{u}_e}{\partial t} \;=\; -\, en(\mathbf{E} \;+\; \mathbf{u}_e \times \mathbf{B}) \;-\; \nabla(nT_e) - \nabla \cdot \left[(p_{e\|} - p_{e\perp})(\mathbf{bb} - \mathbf{I}/3)\right] + \mathbf{F}_e^{coll} \;+\; \mathbf{F}_e^{RF} \;.$$

$$O(nm_e v_{the}^2/L) \qquad\qquad O(\delta^2 n m_e v_{the}^2/L) \qquad O(\delta^3 n m_e v_{the}^2/L)$$

The closure variables to be determined kinetically are

$$(p_{e\|} - p_{e\perp}) \;=\; \frac{m_e}{2} \int d^3\mathbf{v}\; \{3[\mathbf{b} \cdot (\mathbf{v} - \mathbf{u}_e)]^2 - |\mathbf{v} - \mathbf{u}_e|^2\}\; f_{NMe} \;=\; O(\delta^2 n m_e v_{the}^2) \;+\; O(\delta^3 n m_e v_{the}^2) \;,$$

$$\mathbf{F}_e^{coll} \;=\; m_e \int d^3\mathbf{v}\; (\mathbf{v} - \mathbf{u}_e)\; C_{e\iota}(f_e, f_\iota) \;=\; O(\delta^3 n m_e v_{the}^2)$$

and

$$\mathbf{F}_e^{RF} \;=\; m_e \int d^3\mathbf{v}\; (\mathbf{v} - \mathbf{u}_e)\; S^{RF}(f_e) \;=\; O(\delta^3 n m_e v_{the}^2) \;.$$

ELECTRON FLUID TEMPERATURE EQUATION

Keeping $O(\delta^2 n m_e v_{the}^2/L) + O(\delta^3 n m_e v_{the}^2/L)$**:**

$$\frac{3n}{2}\frac{\partial T_e}{\partial t} = -\frac{3n}{2}\mathbf{u}_e \cdot \nabla T_e - nT_e \nabla \cdot \mathbf{u}_e - \nabla \cdot \left(q_{e\|}\mathbf{b} - \frac{5nT_e}{2eB}\mathbf{b} \times \nabla T_e \right) + G_e^{coll} + G_e^{RF} \; .$$

$$O(\delta^2 n m_e v_{the}^3/L) \qquad\qquad O(\delta^3 n m_e v_{the}^3/L)$$

The closure variables to be determined kinetically are

$$q_{e\|} = \frac{m_e}{2}\int d^3\mathbf{v}\; [\mathbf{b}\cdot(\mathbf{v}-\mathbf{u}_e)]\; |\mathbf{v}-\mathbf{u}_e|^2\; f_{NMe} = O(\delta^2 n m_e v_{the}^3) + O(\delta^3 n m_e v_{the}^3),$$

$$G_e^{coll} = \frac{m_e}{2}\int d^3\mathbf{v}\; |\mathbf{v}-\mathbf{u}_e|^2\; C_{e\iota}(f_e, f_\iota) = O(\delta^3 n m_e v_{the}^3/L)$$

and

$$G_e^{RF} = \frac{m_e}{2}\int d^3\mathbf{v}\; |\mathbf{v}-\mathbf{u}_e|^2\; S^{RF}(f_e) = O(\delta^3 n m_e v_{the}^3/L) \; .$$

KINETIC EQUATION FOR THE NON-MAXWELLIAN PART OF THE ELECTRON DISTRIBUTION FUNCTION

In terms of the velocity-space coordinates $(v'_\parallel, v'_\perp, \alpha)$ **in the reference frame of the electron macroscopic flow,**

$$\mathbf{v} \;=\; \mathbf{u}_e(\mathbf{x},t) \;+\; v'_\parallel\;\mathbf{b}(\mathbf{x},t) \;+\; v'_\perp\;[\cos\alpha\;\mathbf{e}_1(\mathbf{x},t)+\sin\alpha\;\mathbf{e}_2(\mathbf{x},t)]\;,$$

the non-Maxwellian part of the electron distribution function can be represented as

$$f_{NMe}(v'_\parallel, v'_\perp, \alpha, \mathbf{x}, t) \;=\; \bar{f}_{NMe}(v'_\parallel, v'_\perp, \mathbf{x}, t) \;+\; \tilde{f}_{NMe}(v'_\parallel, v'_\perp, \alpha, \mathbf{x}, t)$$

with

$$\langle \tilde{f}_{NMe} \rangle_\alpha \equiv (2\pi)^{-1} \oint d\alpha\;\tilde{f}_{NMe} = 0\;.$$

Then, keeping the accuracy of $O(\delta^2 f_{Me}) + O(\delta^3 f_{Me})$**:**

$$\tilde{f}_{NMe} \;=\; f_{Me}\,\frac{m_e v'_\perp}{2eBT_e}\left[\frac{m_e}{T_e}(v'^2_\parallel + v'^2_\perp) - 5\right](\cos\alpha\;\mathbf{e}_2 - \sin\alpha\;\mathbf{e}_1)\cdot\nabla T_e$$

and $\bar{f}_{NMe}$ obeys the following drift-kinetic equation:

$$\frac{\partial \bar{f}_{NMe}}{\partial t} + v'_{\|}\mathbf{b}\cdot\frac{\partial \bar{f}_{NMe}}{\partial \mathbf{x}} + \left(\frac{T_e}{m_e}\mathbf{b}\cdot\nabla\ln n - \frac{v'^2_{\perp}}{2}\mathbf{b}\cdot\nabla\ln B\right)\frac{\partial \bar{f}_{NMe}}{\partial v'_{\|}} + \frac{v'_{\perp}v'_{\|}}{2}\mathbf{b}\cdot\nabla\ln B\,\frac{\partial \bar{f}_{NMe}}{\partial v'_{\perp}} =$$

$$= \Big\{\frac{v'_{\|}}{2T_e}\left[5 - \frac{m_e}{T_e}(v'^2_{\|} + v'^2_{\perp})\right]\mathbf{b}\cdot\nabla T_e + \frac{v'_{\|}}{nT_e}\,\mathbf{b}\cdot\left[\frac{2}{3}\nabla(p_{e\|} - p_{e\perp}) - \big(p_{e\|} - p_{e\perp}\big)\nabla\ln B - \mathbf{F}_e^{coll} - \mathbf{F}_e^{RF}\right] +$$

$$+ \frac{m_e}{6T_e}(2v'^2_{\|} - v'^2_{\perp})\big(\nabla\cdot\mathbf{u}_e - 3\mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)\mathbf{u}_e]\big) + \frac{1}{3nT_e}\left[\frac{m_e}{T_e}(v'^2_{\|} + v'^2_{\perp}) - 3\right]\left[\nabla\cdot(q_{e\|}\mathbf{b}) - G_e^{coll} - G_e^{RF}\right] +$$

$$+ \frac{1}{2eB}\left[\frac{m_e^2}{T_e^2}(v'^4_{\|} + v'^2_{\|}v'^2_{\perp}) - \frac{5m_e}{3T_e}(4v'^2_{\|} + v'^2_{\perp}) + 5\right](\mathbf{b}\times\boldsymbol{\kappa})\cdot\nabla T_e +$$

$$+ \frac{1}{2eB}\left[\frac{m_e^2}{2T_e^2}(v'^2_{\|}v'^2_{\perp} + v'^4_{\perp}) - \frac{5m_e}{6T_e}(2v'^2_{\|} + 5v'^2_{\perp}) + 5\right](\mathbf{b}\times\nabla\ln B)\cdot\nabla T_e +$$

$$+ \frac{m_e}{6eBT_e}(2v'^2_{\|} - v'^2_{\perp})(\mathbf{b}\times\nabla\ln n)\cdot\nabla T_e\ \Big\}f_{Me} +$$

$$+ \langle C_{ee}^{(3)}(f_{Me}, f_{NMe}) + C_{ee}^{(3)}(f_{NMe}, f_{Me})\rangle_{\alpha} + \langle C_{e\iota}^{(3)}(f_{Me}, f_{\iota}) + C_{e\iota}^{(3)}(f_{NMe}, f_{M\iota})\rangle_{\alpha} + \langle S^{RF}(f_{Me})\rangle_{\alpha}\ .$$

With the 1, $v'_{\|}$ and $(v'^2_{\|} + v'^2_{\perp})$ moments of $\bar{f}_{NMe}$ equal to zero, the 1, $v'_{\|}$ and $(v'^2_{\|} + v'^2_{\perp})$ moments of this drift-kinetic equation are satisfied identically.

In polar coordinates, $v'_\parallel = v' \cos\chi,\ \ v'_\perp = v' \sin\chi$:

$$\frac{\partial \bar{f}_{NMe}}{\partial t} \ + \ \cos\chi \left(v'\mathbf{b} \cdot \frac{\partial \bar{f}_{NMe}}{\partial \mathbf{x}} + \frac{T_e}{m_e}\mathbf{b} \cdot \nabla \ln n \ \frac{\partial \bar{f}_{NMe}}{\partial v'} \right) \ - \ \frac{\sin\chi}{v'} \left(\frac{T_e}{m_e}\mathbf{b} \cdot \nabla \ln n - \frac{v'^2}{2}\mathbf{b} \cdot \nabla \ln B \right) \frac{\partial \bar{f}_{NMe}}{\partial \chi} =$$

$$= \Bigg\{ \cos\chi \ \frac{v'}{2T_e} \left(5 - \frac{m_e v'^2}{T_e} \right) \mathbf{b} \cdot \nabla T_e \ + \ \cos\chi \ \frac{v'}{nT_e} \ \mathbf{b} \cdot \left[\frac{2}{3} \nabla (p_{e\parallel} - p_{e\perp}) - \left(p_{e\parallel} - p_{e\perp} \right) \nabla \ln B - \mathbf{F}_e^{coll} - \mathbf{F}_e^{RF} \right] +$$

$$+ \ P_2(\cos\chi) \ \frac{m_e v'^2}{3T_e} \big(\nabla \cdot \mathbf{u}_e - 3\mathbf{b} \cdot [(\mathbf{b} \cdot \nabla)\mathbf{u}_e] \big) \ + \ \frac{1}{3nT_e} \left(\frac{m_e v'^2}{T_e} - 3 \right) \left[\nabla \cdot (q_{e\parallel}\mathbf{b}) - G_e^{coll} - G_e^{RF} \right] \ +$$

$$+ \ \frac{1}{6eB} \left[2P_2(\cos\chi) \frac{m_e v'^2}{T_e} \left(\frac{m_e v'^2}{T_e} - 5 \right) \ + \ \frac{m_e^2 v'^4}{T_e^2} - 10 \frac{m_e v'^2}{T_e} + 15 \right] (\mathbf{b} \times \boldsymbol{\kappa}) \cdot \nabla T_e \ +$$

$$+ \ \frac{1}{6eB} \left[-P_2(\cos\chi) \frac{m_e v'^2}{T_e} \left(\frac{m_e v'^2}{T_e} - 5 \right) \ + \ \frac{m_e^2 v'^4}{T_e^2} - 10 \frac{m_e v'^2}{T_e} + 15 \right] (\mathbf{b} \times \nabla \ln B) \cdot \nabla T_e \ +$$

$$+ \ P_2(\cos\chi) \ \frac{m_e v'^2}{3eBT_e} (\mathbf{b} \times \nabla \ln n) \cdot \nabla T_e \ \Bigg\} f_{Me} \ +$$

$$+ \ \langle C_{ee}^{(3)}(f_{Me}, f_{NMe}) + C_{ee}^{(3)}(f_{NMe}, f_{Me}) \rangle_\alpha \ + \ \langle C_{e\iota}^{(3)}(f_{Me}, f_\iota) + C_{e\iota}^{(3)}(f_{NMe}, f_{M\iota}) \rangle_\alpha \ + \ \langle S^{RF}(f_{Me}) \rangle_\alpha \ .$$

COLLISIONAL TERMS

Based on the complete form of the linearized Fokker-Plank operators and using the electron collision frequency definition

$$\nu_e \;\equiv\; \frac{c^4 e^4 n \ln \Lambda_e}{4\pi m_e^2 v_{the}^3} \;,$$

the gyrophase averaged collision operators that enter in the drift-kinetic equation are as follows:

$$\langle C^{(3)}_{e\iota}(f_{Me}, f_\iota)\rangle_\alpha \;=\; \frac{\nu_e m_e}{m_\iota}\left(\frac{T_i}{T_e} - 1\right) f_{Me}(v') \left[\frac{v_{the}}{v'}\;\phi\left(\frac{v'}{v_{th\iota}}\right) - \frac{4\pi v_{the}^3}{n}\; f_{M\iota}(v')\right] \;+$$

$$+\;\frac{\nu_e j_\| v_{the}}{e n v_{th\iota}^3}\; f_{Me}(v')\;\left[\frac{v_{th\iota}}{v'}\;\xi\left(\frac{v'}{v_{th\iota}}\right) - \frac{4\pi v_{th\iota}^3}{n}\; f_{M\iota}(v')\right]\; v'\cos\chi$$

where

$$\phi(x) \;=\; \frac{2}{(2\pi)^{1/2}}\int_0^x dt\;\;\exp(-t^2/2) \qquad \textbf{and} \qquad \xi(x) \;=\; \frac{1}{x^2}\left[\phi(x) - x\frac{d\phi(x)}{dx}\right]\;.$$

$\langle C_{ee}^{(3)}(f_{Me}, f_{NMe}) + C_{ee}^{(3)}(f_{NMe}, f_{Me})\rangle_\alpha + \langle C_{e\iota}^{(3)}(f_{NMe}, f_{M\iota})\rangle_\alpha \;\equiv\; \mathcal{C}[\bar{f}_{NMe}]$ **is Legendre diagonal:**

$$\mathcal{C}\left[\sum_{l=0}^{\infty} f_l(v')P_l(\cos\chi)\right] \;=\; \sum_{l=0}^{\infty} P_l(\cos\chi)\; \mathcal{C}_l[f_l(v')]$$

with

$$\begin{aligned}
C_l[f_l(v')] \;=\; & \frac{\nu_e}{n} f_{Me}(v')\Big\{4\pi v_{the}^3 f_l(v') \;-\; v_{the}\Phi_l^R[f_l(v')] \;+\; v_{the}^{-1}\Xi_l^R[f_l(v')]\Big\} \;+ \\
& + \frac{\nu_e v_{the}^3}{v'^2}\frac{\partial}{\partial v'}\left\{\xi\left(\frac{v'}{v_{the}}\right)\left[v'\frac{\partial f_l(v')}{\partial v'} \;+\; \frac{v'^2}{v_{the}^2}\; f_l(v')\right]\right\} \;- \\
& - \;\frac{\nu_e l(l+1)v_{the}^3}{2v'^3}\left[\phi\left(\frac{v'}{v_{the}}\right) - \xi\left(\frac{v'}{v_{the}}\right) + \phi\left(\frac{v'}{v_{th\iota}}\right) - \xi\left(\frac{v'}{v_{th\iota}}\right)\right] f_l(v')
\end{aligned}$$

and

$$\frac{1}{v'^2}\frac{\partial}{\partial v'}\left\{v'^2\frac{\partial\Phi_l^R[f_l(v')]}{\partial v'}\right\} \;-\; \frac{l(l+1)}{v'^2}\Phi_l^R[f_l(v')] \;=\; -4\pi f_l(v')\;,$$

$$\Xi_l^R[f_l(v')] \;=\; v'^2\frac{\partial^2\Psi_l^R[f_l(v')]}{\partial v'^2}\;,$$

$$\frac{1}{v'^2}\frac{\partial}{\partial v'}\left\{v'^2\frac{\partial\Psi_l^R[f_l(v')]}{\partial v'}\right\} \;-\; \frac{l(l+1)}{v'^2}\Psi_l^R[f_l(v')] \;=\; \Phi_l^R[f_l(v')]\;.$$

The collisional moments in the fluid and drift-kinetic equations are:

$$\mathbf{F}_e^{coll} \;=\; \frac{2m_e\nu_e}{3(2\pi)^{1/2}e}\;\mathbf{j} \;-\; \frac{m_e\nu_e n}{(2\pi)^{1/2}eB}\;(\mathbf{b}\times\nabla T_e) \;-\; \frac{4\pi}{3}m_e\nu_e v_{the}^3\int_0^\infty dv'\;\bar{f}_{NMe,l=1}(v')\;\mathbf{b} \;=\; O\left(\frac{\delta^3 n m_e v_{the}^2}{L}\right)$$

$$G_e^{coll} \;=\; \frac{2m_e\nu_e n}{(2\pi)^{1/2}m_\iota}\;(T_\iota - T_e) \;=\; O\left(\frac{\delta^3 n m_e v_{the}^3}{L}\right)\;.$$

APPLICATION: STATIONARY AXISYMMETRIC SYSTEM

$$\nabla \cdot \mathbf{B} = 0 \ , \qquad \mathbf{j} = \nabla \times \mathbf{B} \ , \qquad \mathbf{E} = -\nabla\Phi - V_0 \nabla\varphi$$

$$\nabla \cdot (n\mathbf{u}_\iota) = \nabla \cdot (n\mathbf{u}_e) = 0 \ , \qquad \mathbf{u}_e = \mathbf{u}_\iota - \frac{1}{en}\mathbf{j}$$

$$\mathbf{b} \cdot \nabla T_e \ = \ O(\delta^2 \ T_e/L) \ , \qquad \mathbf{b} \cdot \nabla T_\iota \ = \ O(\delta^2 \ T_\iota/L) \ , \qquad T_e - T_\iota \ll T_e$$

$$en(\mathbf{E} \ + \ \mathbf{u}_e \times \mathbf{B}) \ + \ \nabla(nT_e) + \nabla \cdot \Big[(p_{e\parallel} - p_{e\perp})(\mathbf{bb} - \mathbf{I}/3)\Big] - \mathbf{F}_e^{coll} \ = \ 0$$

$$-en(\mathbf{E} \ + \ \mathbf{u}_\iota \times \mathbf{B}) \ + \ \nabla(nT_\iota) \ = \ O(\delta^2 \ T_\iota/L)$$

$$\frac{3n}{2}\mathbf{u}_e \cdot \nabla T_e \ + \ nT_e \nabla \cdot \mathbf{u}_e \ + \ \nabla \cdot \left(q_{e\parallel}\mathbf{b} \ - \ \frac{5nT_e}{2eB}\mathbf{b} \times \nabla T_e\right) = \ 0$$

EXACT STATIONARY RELATIONS FROM CONTINUITY AND AXISYMMETRY

$$\mathbf{B} = \nabla\psi \times \nabla\varphi + RB_\varphi \nabla\varphi$$

$$\mathbf{j} = \nabla(RB_\varphi) \times \nabla\varphi - \Delta^*\psi \nabla\varphi$$

$$\mathbf{u}_\iota = \frac{1}{n}\nabla\Psi_\iota \times \nabla\varphi + Ru_{\iota\varphi}\nabla\varphi$$

$$\mathbf{u}_e = \frac{1}{n}\nabla\Psi_e \times \nabla\varphi + Ru_{e\varphi}\nabla\varphi$$

$$RB_\varphi = e(\Psi_\iota - \Psi_e)$$

$$-\Delta^*\psi = enR(u_{\iota\varphi} - u_{e\varphi})$$

LOWEST-ORDER STATIONARY FLUID RELATIONS

The lowest-order stationary fluid system (valid on the MHD time scale $t \lesssim \delta^{-1}\Omega_{c\iota}^{-1}$) yields the well known relations:

$$n = N^{(0)}(\psi), \quad T_s = T_s^{(0)}(\psi), \quad \Phi = \Phi^{(1)}(\psi) = O(T_s/e), \quad \Psi_s = \Psi_s^{(1)}(\psi) = O(\delta n v_{th\iota} L^2)$$

$$RB_\varphi = (RB_\varphi)^{(0)}(\psi) = e[\Psi_\iota^{(1)}(\psi) - \Psi_e^{(1)}(\psi)] \equiv I(\psi)$$

$$\mathbf{u}_s = \mathbf{u}_s^{(1)} = \frac{1}{N^{(0)}}\frac{d\Psi_s^{(1)}}{d\psi}\mathbf{B} + R^2\left[\frac{d\Phi^{(1)}}{d\psi} + \frac{1}{e_s N^{(0)}}\frac{d(N^{(0)}T_s^{(0)})}{d\psi}\right]\nabla\varphi \equiv U_s(\psi)\mathbf{B} + R^2\Omega_s(\psi)\nabla\varphi = O(\delta v_{th\iota})$$

From these, it follows that:

$$\nabla\psi\cdot(\mathbf{b}\times\boldsymbol{\kappa}) = \nabla\psi\cdot(\mathbf{b}\times\nabla\ln B) = I(\psi)\ \mathbf{b}\cdot\nabla\ln B$$

$$\nabla\cdot\mathbf{u}_s = 0$$

$$\mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)\mathbf{u}_s] = U_s(\psi)\ \mathbf{b}\cdot\nabla\ln B$$

HIGHER-ORDER STATIONARY ELECTRON FLUID RELATIONS

Keeping our highest accuracy of $O(\delta^3 n m_e v_{the}^2/L)$, the parallel component of the stationary electron momentum equation yields

$$\mathbf{b}\cdot\left[\frac{2}{3}\nabla(p_{e\|}-p_{e\perp})-\left(p_{e\|}-p_{e\perp}\right)\nabla\ln B-\mathbf{F}_e^{coll}\right] \;=\; N^{(0)}T_e^{(0)}\;\mathbf{b}\cdot\nabla\left(\frac{e\Phi}{T_e^{(0)}}-\frac{n}{N^{(0)}}-\frac{T_e}{T_e^{(0)}}\right)\;+\;\frac{eV_0N^{(0)}I}{BR^2}$$

and keeping $O(\delta^3 n m_e v_{the}^3/L)$, the stationary electron temperature equation yields

$$\nabla\cdot\left(q_{e\|}\mathbf{b}\right) \;=\; \nabla\cdot\left(\frac{5N^{(0)}T_e^{(0)}}{2eB}\mathbf{b}\times\nabla T_e^{(0)}\right) \;=\; \frac{5N^{(0)}T_e^{(0)}I}{eB}\;\frac{dT_e^{(0)}}{d\psi}\;\mathbf{b}\cdot\nabla\ln B\;.$$

Notice that, within this highest available accuracy, the stationary electron temperature equation does not provide any information on the higher-order correction $T_e(\mathbf{x})-T_e^{(0)}(\psi)$!

STATIONARY DRIFT-KINETIC EQUATION

Using the previous stationary fluid results and calling $g \equiv \bar{f}_{NMe}/f_{Me}^{(0)} = \Sigma_{l=0}^{\infty} g_l \; P_l(\cos\chi)$, the stationary electron drift-kinetic equation can be written as:

$$
\begin{aligned}
& v' \left(\cos\chi \; \mathbf{b} \cdot \frac{\partial g}{\partial \mathbf{x}} \; + \; \frac{1}{2} \mathbf{b} \cdot \nabla \ln B \;\; \sin\chi \; \frac{\partial g}{\partial \chi} \right) \; = \\
& = \; v' \cos\chi \left[\mathbf{b} \cdot \nabla \left(\frac{e\Phi}{T_e^{(0)}} - \frac{n}{N^{(0)}} \right) \; + \; \left(\frac{3}{2} - \frac{m_e v'^2}{2T_e^{(0)}} \right) \mathbf{b} \cdot \nabla \left(\frac{T_e}{T_e^{(0)}} \right) \; + \; \frac{eV_0 I}{T_e^{(0)} B R^2} \right] - \\
& - \; \left\{ P_2(\cos\chi) \; \frac{m_e v'^2}{T_e^{(0)}} \; U_e B \; + \; \left[2 + P_2(\cos\chi) \right] \frac{m_e v'^2}{3T_e^{(0)}} \left(\frac{5}{2} - \frac{2m_e v'^2}{T_e^{(0)}} \right) \frac{I}{eB} \; \frac{dT_e^{(0)}}{d\psi} \right\} \; \mathbf{b} \cdot \nabla \ln B \; + \\
& + \; v' \cos\chi \; \mathcal{D}_1 \; + \; \hat{\mathcal{C}}[g]
\end{aligned}
$$

where

$$
\mathcal{D}_1 \; \equiv \; \frac{\nu_e j_\parallel v_{the}}{e n v_{th\iota}^3} \; \left[\frac{v_{th\iota}}{v'} \; \xi \left(\frac{v'}{v_{th\iota}} \right) - \frac{4\pi v_{th\iota}^3}{n} \; f_{M\iota}(v') \right]
$$

and

$$
\hat{\mathcal{C}}[g] \; \equiv \; \frac{1}{f_{Me}^{(0)}} \; \mathcal{C}\left[g f_{Me}^{(0)} \right] \; \equiv \; \sum_{l=0}^{\infty} P_l(\cos\chi) \; \hat{\mathcal{C}}_l[g_l]
$$

Within $O(\delta^2)$, the above stationary drift-kinetic equation has the particular solution

$$g \;=\; g_p + O(\delta^3) \;=\; \frac{e\Big(\Phi - \Phi^{(1)}\Big)}{T_e^{(0)}} \;-\; \frac{n - N^{(0)}}{N^{(0)}} \;+\; \left(\frac{3}{2} - \frac{m_e v'^2}{2T_e^{(0)}}\right)\frac{T_e - T_e^{(0)}}{T_e^{(0)}} \;+$$

$$+\; v' \cos\chi \left[- \frac{m_e U_e B}{T_e^{(0)}} \;+\; \frac{m_e I}{eBT_e^{(0)}}\left(\frac{5}{2} - \frac{2m_e v'^2}{T_e^{(0)}}\right)\frac{dT_e^{(0)}}{d\psi}\right] \;+\; O(\delta^3) \;\equiv\; g_{p0} \;+\; \cos\chi\; g_{p1} \;+\; O(\delta^3)\;.$$

Therefore, calling $h \equiv g - g_p$ and using the fact that $\hat{\mathcal{C}}[g_{p0}] = 0$, one gets

$$v'\left(\cos\chi\;\mathbf{b}\cdot\frac{\partial h}{\partial \mathbf{x}} \;+\; \frac{1}{2}\mathbf{b}\cdot\nabla\ln B\;\;\sin\chi\;\frac{\partial h}{\partial \chi}\right) \;=\; v'\cos\chi\left(\frac{eV_0 I}{T_e^{(0)}BR^2} \;+\; \mathcal{D}_1\right) \;+\; \hat{\mathcal{C}}[h \;+\; \cos\chi\; g_{p1}]$$

and, changing variables from $\mathbf{x}, v', \chi$ to $\mathbf{x}, v', \lambda$ with $\lambda(\mathbf{x},\chi) = \sin^2\chi\; B_{max}(\psi)/B(\mathbf{x})$:

$$v'_\parallel\;\mathbf{b}\cdot\frac{\partial h(\mathbf{x}, v', \lambda)}{\partial \mathbf{x}} \;=\; v'_\parallel\left[\frac{eV_0 I}{T_e^{(0)}BR^2} \;+\; \mathcal{D}_1(\mathbf{x}, v')\right] \;+\; \hat{\mathcal{C}}[h \;+\; (v'_\parallel/v')\; g_{p1}]$$

where

$$v'_\parallel(\mathbf{x}, v', \lambda) \;=\; \pm v'(1 - \lambda B/B_{max})^{1/2} \qquad \textbf{and} \qquad 0 \le \lambda \le B_{max}/B\;.$$

The drift-kinetic equation for $h(\mathbf{x}, v', \lambda)$ is in the conventional form of the banana regime neoclassical theory. Solving perturbatively:

$$h(\mathbf{x}, v', \lambda) \;=\; \sigma(v'_\|)\; H(1-\lambda)\; K(\psi, v', \lambda) \;+\; h^{(3)}(\mathbf{x}, v', \lambda)\;,$$

$$v'_\| \;\mathbf{b}\cdot\frac{\partial h^{(3)}(\mathbf{x}, v', \lambda)}{\partial \mathbf{x}} \;=\; v'_\| \left[\frac{eV_0 I}{T_e^{(0)} B R^2} \;+\; \mathcal{D}_1(\mathbf{x}, v')\right] \;+\; \hat{\mathcal{C}}\Big[\sigma(v'_\|)\; H(1-\lambda)\; K(\psi, v', \lambda) \;+\; (v'_\|/v')\; g_{p1}\Big]$$

which has the integrability constraint

$$\oint \frac{dl}{v'_\|}\; \hat{\mathcal{C}}\Big[\sigma(v'_\|)\; H(1-\lambda)\; K(\psi, v', \lambda) \;+\; (v'_\|/v')\; g_{p1}\Big] \;=\; -\oint dl \left[\frac{eV_0 I}{T_e^{(0)} B R^2} \;+\; \mathcal{D}_1(\mathbf{x}, v')\right]$$

and the solution of this Spitzer problem determines the function $K(\psi, v', \lambda)$.

$h^{(3)}$ is an even function of $v'_\|$, so the odd part of h is completely specified by its lowest-order solution $\sigma(v'_\|)\; H(1-\lambda)\; K(\psi, v', \lambda) \;=\; O(\delta^2)$.

The drift-kinetic solution for $\bar{f}_{NMe} \ = \ f_{Me}^{(0)} \ g \ = \ f_{Me}^{(0)} \ (g_p + h)$ **in the reference frame of the macroscopic flow must satisfy**

$$\int d^3\mathbf{v}' \ v'_{\|} \ \bar{f}_{NMe} \ = \ 0 \ , \qquad \int d^3\mathbf{v}' \ \bar{f}_{NMe} \ = \ 0 \ , \qquad \int d^3\mathbf{v}' \ v'^2 \ \bar{f}_{NMe} \ = \ 0 \ .$$

Imposing these conditions:

$$N^{(0)}(\psi) \ U_e(\psi) \ B_{max}(\psi) \ = \ 2\pi \ \int_0^\infty dv' \ v'^3 f_{Me}^{(0)}(\psi, v') \ \int_0^1 d\lambda \ K(\psi, v', \lambda) \ ,$$

$$\frac{e\Big(\Phi - \Phi^{(1)}(\psi)\Big)}{T_e^{(0)}(\psi)} - \frac{n - N^{(0)}(\psi)}{N^{(0)}(\psi)} \ = \ O(\delta^3) \qquad \textbf{and} \qquad \frac{T_e - T_e^{(0)}(\psi)}{T_e^{(0)}(\psi)} \ = \ O(\delta^3) \ .$$

Therefore the even part of the distribution function is $\bar{f}_{NMe}^{even} \ = \ O(\delta^3 \ f_{Me}^{(0)})$ **, which implies** $(p_{e\|} - p_{e\perp})/(N^{(0)} \ T_e^{(0)}) \ = \ O(\delta^3)$ **consistent with the parallel component of the electron fluid momentum equation.**

ODD PARALLEL CLOSURES

The odd parallel closures

$$q_{e\|} \;=\; (m_e/2)\int d^3\mathbf{v}'\; v'^2\; v'_\| \;\bar{f}_{NMe}$$

and

$$F^{coll}_{e\|} \;=\; \frac{2m_e\nu_e}{3(2\pi)^{1/2}e}\; j_\| \;-\; m_e\nu_e v^3_{the}\int d^3\mathbf{v}'\;\frac{v'_\|}{v'^3}\;\bar{f}_{NMe}$$

can be evaluated once the function $K(\psi, v', \lambda)$ **is known:**

$$q_{e\|} \;=\; -\;\frac{5N^{(0)}T_e^{(0)}}{2}\left(\frac{I}{eB}\frac{dT_e^{(0)}}{d\psi} + U_e B\right) \;+\; \frac{\pi m_e B}{B_{max}}\;\int_0^\infty dv'\;\; v'^5 f^{(0)}_{Me}(\psi, v')\;\;\int_0^1 d\lambda\;\, K(\psi, v', \lambda)$$

and

$$F^{coll}_{e\|} \;=\; \frac{2m_e\nu_e}{3(2\pi)^{1/2}}\left(\frac{j_\|}{e} - N^{(0)}U_e B + \frac{3N^{(0)}I}{2eB}\frac{dT_e^{(0)}}{d\psi}\right) \;-\; \frac{2\pi m_e\nu_e v^3_{the} B}{B_{max}}\;\int_0^\infty dv'\;\, f^{(0)}_{Me}(\psi, v')\;\;\int_0^1 d\lambda\;\, K(\psi, v', \lambda)$$

CLOSURE PROBLEM FOR THE PRESSURE ANISOTROPY $(p_{e\|} - p_{e\perp})$

With a Legendre series expansion, $h(\mathbf{x}, v', \chi) = \Sigma_{l=0}^{\infty} h_l(\mathbf{x}, v') P_l(\cos\chi)$, the $l = 2$ projection of the drift-kinetic equation for $h(\mathbf{x}, v', \chi)$ can be expressed after algebraic elimination of the electric potential using the fluid momentum equation as:

$$B^{3/2}\,\mathbf{b}\cdot\frac{\partial}{\partial \mathbf{x}}\left\{B^{-3/2}\left[\frac{2}{5}h_2(\mathbf{x}, v') - \frac{2(p_{e\|} - p_{e\perp})}{3N^{(0)}T_e^{(0)}}\right]\right\} + \mathbf{b}\cdot\frac{\partial k_0(\mathbf{x}, v')}{\partial \mathbf{x}} =$$

$$= \left(\frac{5}{2} - \frac{m_e v'^2}{2T_e^{(0)}}\right)\mathbf{b}\cdot\nabla\left(\frac{T_e}{2T_e^{(0)}}\right) - \frac{F_e^{coll}}{N^{(0)}T_e^{(0)}} + \mathcal{D}_1(\mathbf{x}, v') + \frac{1}{v'}\hat{\mathcal{C}}_1[h_1 + g_{p1}] \;,$$

where $k_0(\mathbf{x}, v') = O(\delta^3)$ is an unknown function that must satisfy

$$\int_0^\infty dv'\; v'^2\; k_0(\mathbf{x}, v') = 0 \qquad \textbf{and} \qquad \int_0^\infty dv'\; v'^4\; k_0(\mathbf{x}, v') = 0 \;,$$

and

$$h_1(\mathbf{x}, v') = \frac{3B(\mathbf{x})}{2B_{max}(\psi)}\int_0^1 d\lambda\; K(\psi, v', \lambda) \;.$$

The pressure anisotropy is given by $(p_{e\|} - p_{e\perp}) = (4\pi m_e/5)\int_0^\infty dv'\; v'^4\; h_2\; f_{Me}^{(0)}$, but the $\int_0^\infty dv'\; v'^4\; f_{Me}^{(0)}$ moment of the above equation results in an identity!

The conclusion is reached that, within the available accuracy of $O(\delta^3)$, the stationary and axisymmetric drift-kinetic equation does not contain information on the pressure anisotropy moment $(p_{e\|} - p_{e\perp})(\mathbf{x}) = O(\delta^3 n T_e)$.

This is consistent with the fluid moment equation for $(p_{e\|} - p_{e\perp})$ which, in the stationary and axisymmetric case and within $O(\delta^3)$ accuracy, reduces to

$$\nabla \cdot \left[(2q_{eB\|} - q_{eT\|})\mathbf{b}\right] \; + \; 3q_{eT\|}\mathbf{b} \cdot \nabla \ln B \; + \; \nabla \cdot \left(\frac{N^{(0)}T_e^{(0)}}{eB}\mathbf{b} \times \nabla T_e^{(0)}\right) \; + \; \frac{3N^{(0)}T_e^{(0)}}{eB}\left(\mathbf{b} \times \nabla T_e^{(0)}\right) \cdot \boldsymbol{\kappa} = 0.$$

So, the determination of $(p_{e\|} - p_{e\perp})(\mathbf{x})$ as well as $T_e(\mathbf{x}) - T_e^{(0)}(\psi)$ in a stationary and axixymmetric system requires carrying the analysis at least to $O(\delta^4)$. This would necesitate a drift-kinetic equation accurate to the second order of the electron gyroradius and including the quadratic parts of the collision operators, something deemed impractical.

CONCLUSIONS

A closed fluid and drift-kinetic electron system for slow-MHD including the interaction with ECCD sources has been derived. It is accurate to the third order in a small ion gyroradius and large parallel collisional mean free path expansion, thus providing a rigorous and realistic description of fusion-relevant plasmas.

This system is compatible with the neoclassical theory for electrons in the banana regime and contains all the desired neoclassical features so it can be confidently used to simulate NTM dynamics.

A consistent NTM simulation must be carried out dynamically and in full three-dimensional geometry. Shortcuts based on stationary and axisymmetric considerations cannot provide a consistent closure of the pressure tensor in the realistic electron-banana collisionality regime. A consistent stationary and axisymmetric determination of the poloidal flow and the odd parallel closures is possible, but this is not sufficient because the proper evaluation of the parallel electric field necessitates the pressure tensor.