# Sawtooth with Energetic Ion Component: Status & Plans

Josh Breslau

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# Motivation

- M3D and NIMROD have previously conducted a successful nonlinear benchmark for the CEMM SciDAC based on the sawtooth cycle in CDX-U with a simplified transport model.
  - Results agree with each other but not with experiment.
  - Better fidelity to experiment should yield better validation.
    - Replace current source with loop voltage.
    - Replace pressure source with ohmic heating.
    - Use a much more realistic profile for $\kappa_{\perp}$.
    - Allow resistivity to track evolving temperature profile.
    - Use constant Prandtl number.
- Beginning with an analytically specified equilibrium will make it possible to publish the benchmark as a standard test problem available to other nonlinear MHD codes.
- Adding hot ions extends the result to more interesting devices, extends benchmarking database, allows comparisons with coupled codes.

# M3D-K Fluid/Particle Hybrid Model

**MHD Equations**

$$nM_i\left(\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\bullet\nabla\mathbf{V}\right)+\nabla p+\nabla\bullet\mathbf{P}_h=\mathbf{J}\times\mathbf{B}$$

$$\frac{\partial \mathbf{B}}{\partial t}=-\nabla\times\mathbf{E},\qquad \mathbf{J}=\nabla\times\mathbf{B},\qquad \mathbf{E}=-\mathbf{V}\times\mathbf{B}+\eta\mathbf{J}$$

$$\frac{\partial p}{\partial t}+\mathbf{V}\bullet\nabla p+\gamma p\nabla\bullet\mathbf{V}=S$$

**CGL Pressure Tensor**

$$\mathbf{P}_h=P_\perp\mathbf{I}+(P_\parallel-P_\perp)\mathbf{bb}$$

$$f=\sum_i\delta(R-R_i)\delta(v_\parallel-v_{\parallel,i})\delta(\mu-\mu_i)$$

**Gyrokinetic Equations**

$$\frac{d\mathbf{R}}{dt}=\frac{1}{B^{**}}\left[v_\parallel\mathbf{B}^*-\mathbf{b_0}\times\left(<\mathbf{E}>-\frac{\mu}{q}\nabla(B_0+<\delta B>\right)\right]$$

$$M_i\frac{dv_\parallel}{dt}=\frac{q}{B^{**}}\mathbf{B}^*\bullet\left(<\mathbf{E}>-\frac{\mu}{q}\nabla(B_0+<\delta B>\right)$$

$$\mathbf{B}^*\equiv\mathbf{B}_0+<\delta\mathbf{B}>+\frac{M_iv_\parallel}{q}\nabla\times\mathbf{b_0},\qquad B^{**}\equiv\mathbf{B}^*\bullet\mathbf{b}_0$$

# Gyrokinetic equation (for particles) can use either $\delta f$ or full-$f$ method

in $\delta f$, distribution function is sum of equilibrium + perturbed

$$f = f_0 + \delta f = f_0 + g \times w$$

"weight function" evolves in time

$$\frac{dw}{dt} = -\left(\frac{f}{g} - w\right)\frac{1}{f_0}\frac{df_0}{dt}$$

equilibrium distribution is a function of the adiabatic invariants

$$f_0 = f_0(P_\phi, E, \mu)$$

equilibrium evolves (slowly) as the adiabatic invariants for each particle change due to detailed particle trajectory motions

$$\frac{df_0}{dt} = \frac{dP_\phi}{dt}\frac{\partial f_0}{\partial P_\phi} + \frac{dE}{dt}\frac{\partial f_0}{\partial E}$$

$$\frac{dE}{dt} = ev_d < \mathbf{E} > + M_i \mu \frac{d}{dt} < \delta B >$$

$$\frac{dP_\phi}{dt} = \left(\frac{d\mathbf{R}}{dt}\right)_1 \bullet \nabla P_\phi + \left(\frac{dv_\parallel}{dt}\right)_1 \frac{\partial P_\phi}{\partial v_\parallel}$$

# M3D Verification and Validation

- Good agreement between M3D and NIMROD for CDX-U sawteeth simulations;
- Good agreement between M3D-K, NOVA2, and NIMROD for energetic particle stabilization of internal kink and excitation of fishbone;
- M3D-K results of beam-driven TAEs are consistent with NSTX observations: mode frequency and its chirping, mode saturation time scale.

# Specification of Analytic Equilibrium

| Quantity | Value |
|---|---|
| Major radius $R_0$ | 0.341 m |
| Minor radius $a$ | 0.247 m (aspect ratio = 1.38) |
| Ellipticity $\kappa$ | 1.35 |
| Triangularity $\delta$ | 0.25 |
| Central temperature ($T_e$=$T_i$) | 100 eV |
| Normalized central pressure $\mu_0 p_0$ | $7.5 \times 10^{-4}$ (implies $n_0 = 1.86 \times 10^{19}$ $m^{-3}$) |
| $\alpha$ Parameter in pressure equation* | 0.1 |
| Vacuum value $g_0$ of $R \cdot B_T$ | 0.04252 T·m |
| Effective ion charge $Z_{EFF}$ | 2.0 |
| Loop voltage $V_L$ | 3.1741 V (implies $q_0 \approx 0.82$) |

$$*p(\psi) = p_0\left[\alpha\tilde{\psi} + (1-\alpha)\tilde{\psi}^2\right], \text{ where } \tilde{\psi} \equiv \frac{\psi - \psi_{\text{limiter}}}{\psi_{\text{axis}} - \psi_{\text{limiter}}}.$$

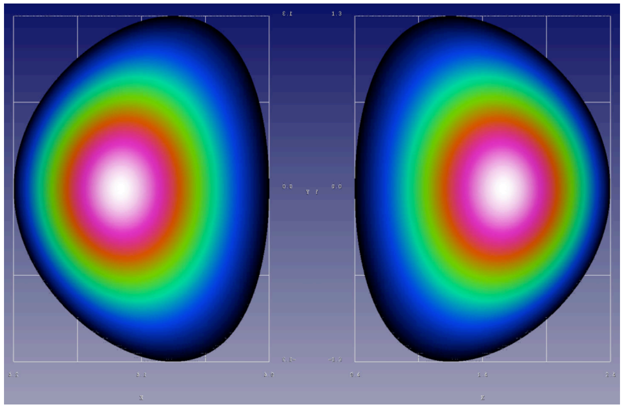
Use equilibrium code to solve Grad-Shafranov equation, with profile of heat conduction coefficient $\chi$ computed self-consistently to keep temperature constant given profile, energy supplied by applied $V_L$.

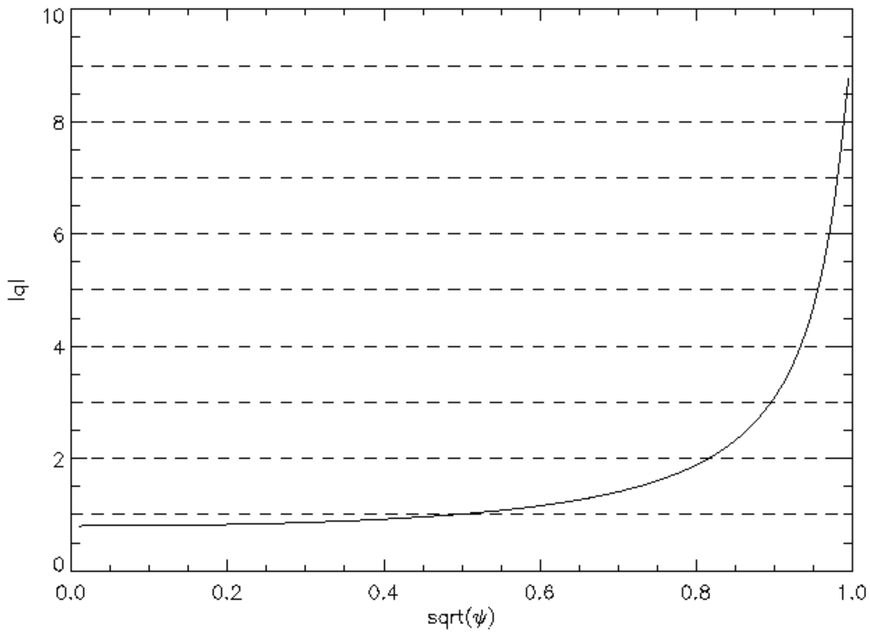
# Form of New Equilibrium

$$R(\theta) = R_0 + a\cos\left[\theta + \delta\sin(\theta)\right]$$
$$z(\theta) = a\kappa\sin(\theta)$$

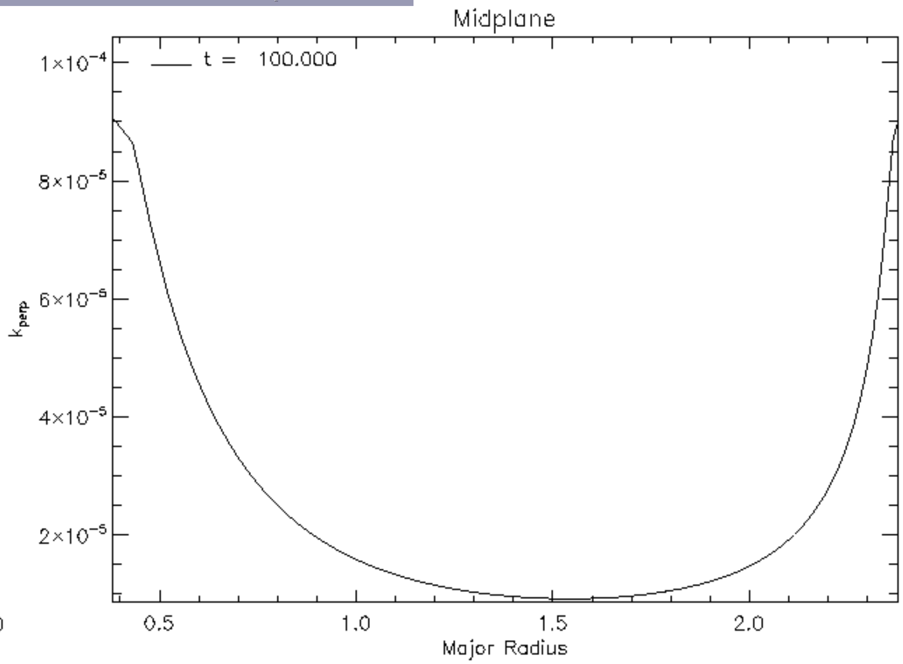
$$T(\psi) = T_0\tilde{\psi},$$

$$n(\psi) = \frac{p}{2k_BT} = \frac{p_0}{2k_BT_0}\left[\alpha + (1-\alpha)\tilde{\psi}\right]$$






$q_{min}$ = 0.8023




Minimum value: $9.21 \times 10^{-6}$

# Transport Coefficients

- Evolving Spitzer resistivity $\eta(\boldsymbol{x},t) \propto T^{-3/2}$ with cutoff 100x initial central value; initial central $S$ = $5.45 \times 10^4$.

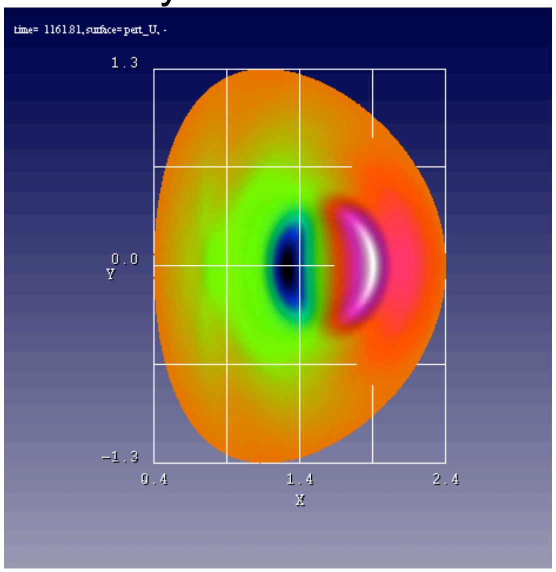
- Constant Prandtl number 10 (evolving axisymmetric viscosity).

- Perpendicular heat diffusivity $\kappa_\perp$ read from self-consistent steady state computed with equilibrium code; central value renormalized to about 2.03 $m^2/s$ to maintain steady-state.

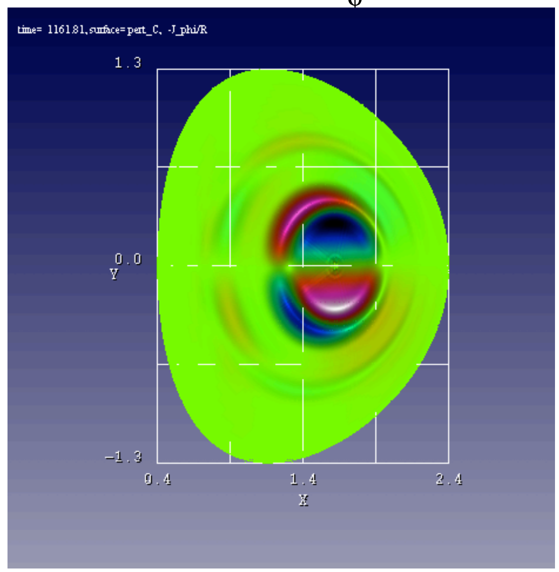
- Parallel heat conduction: ($v_{Te}$ = 6 $v_A$).

# $n$=1 eigenmode

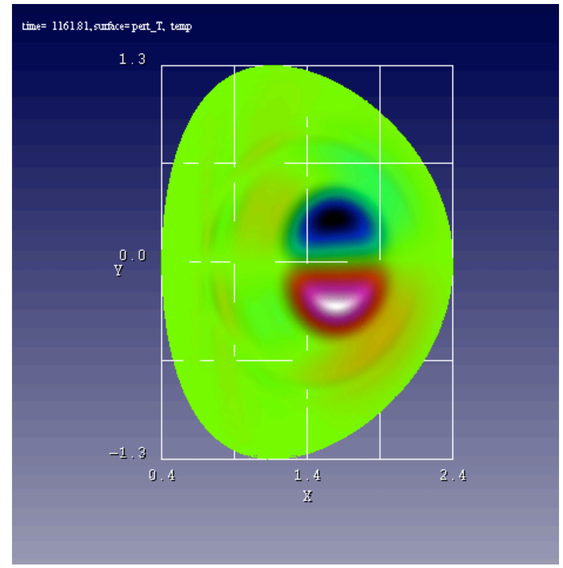
Velocity stream function U




$C = -RJ_{\phi}$

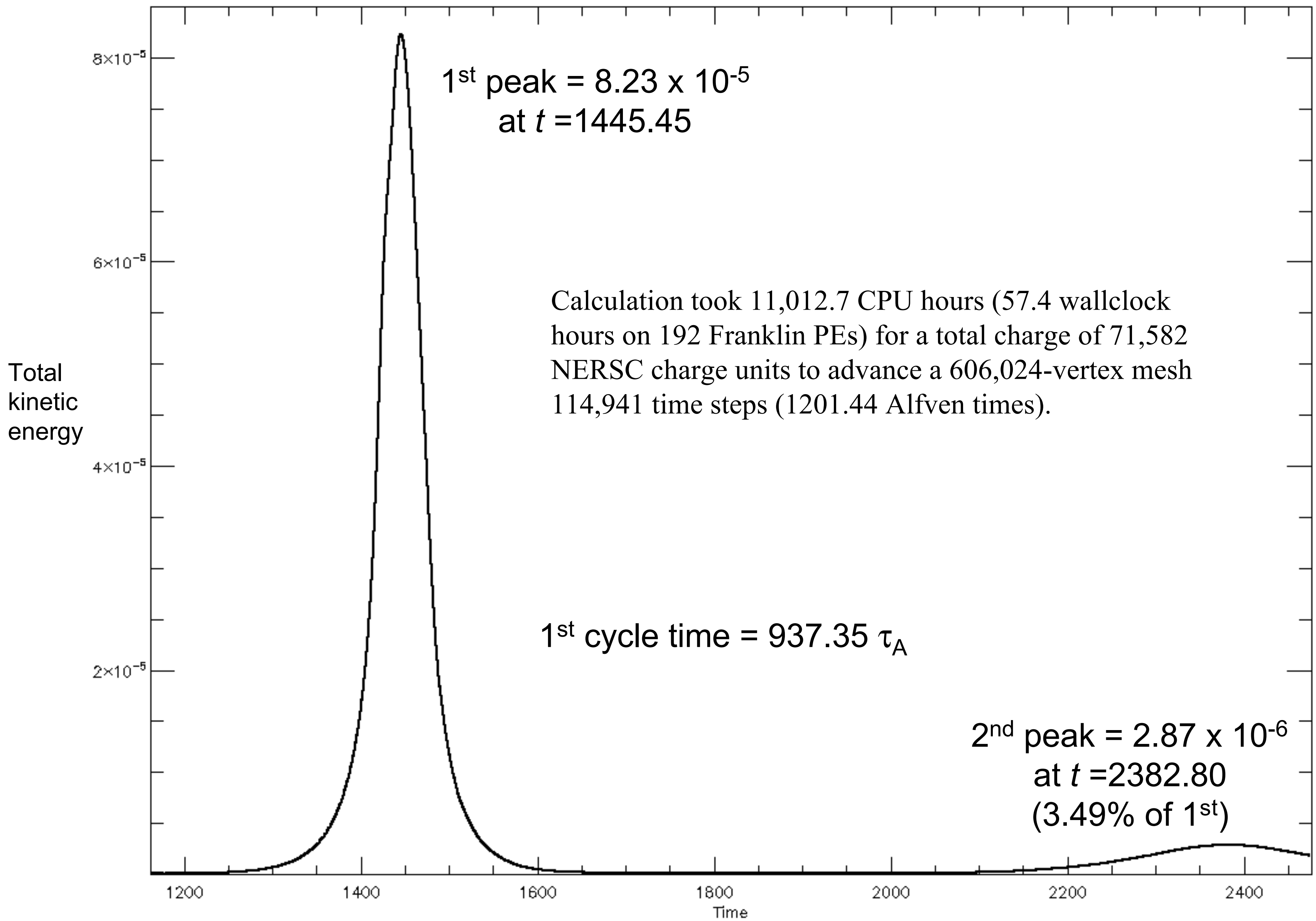



Temperature

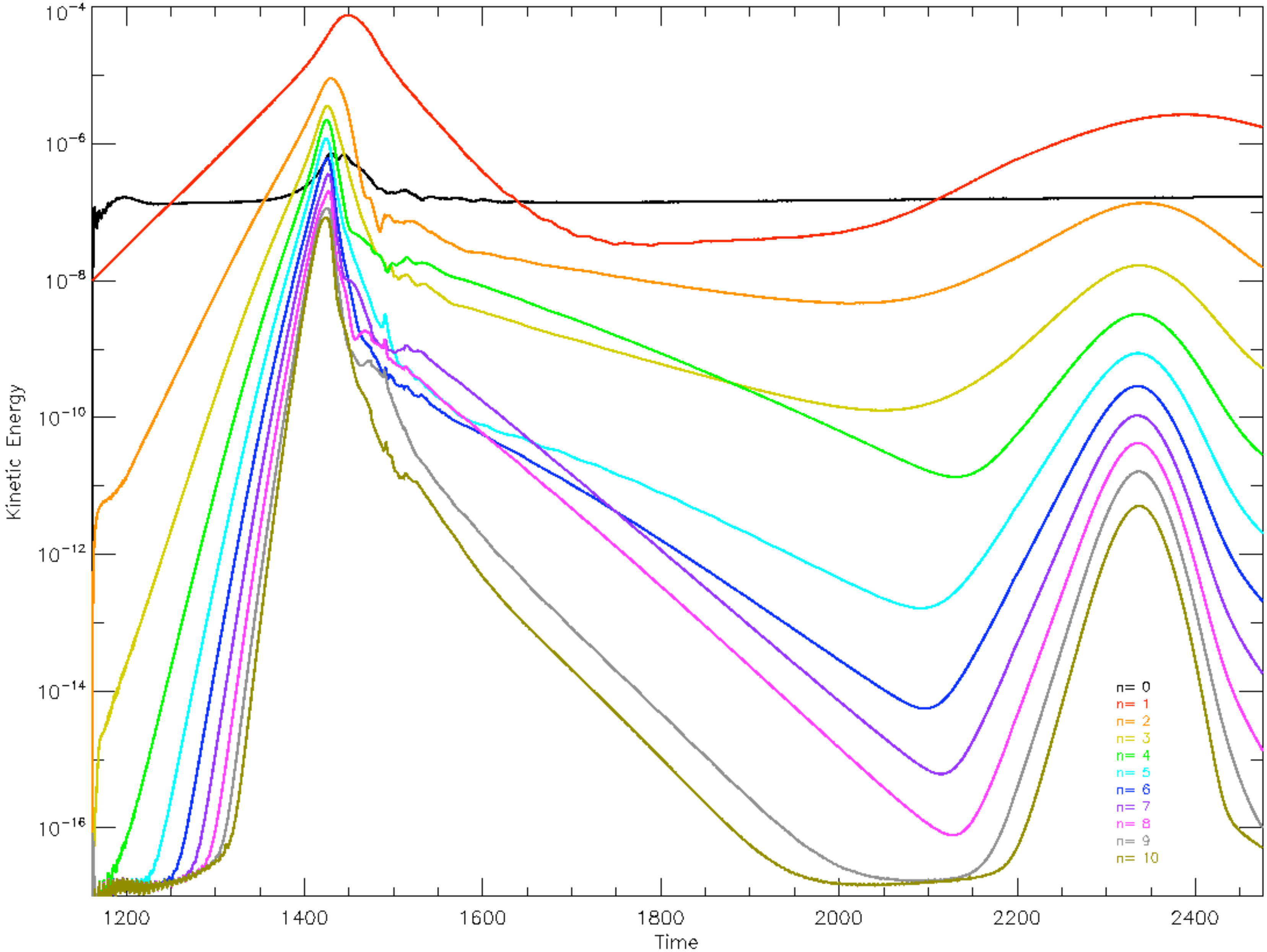



1,1 mode; $\gamma\tau_A \approx (1.5402 \pm 0.0005) \times 10^{-2}$

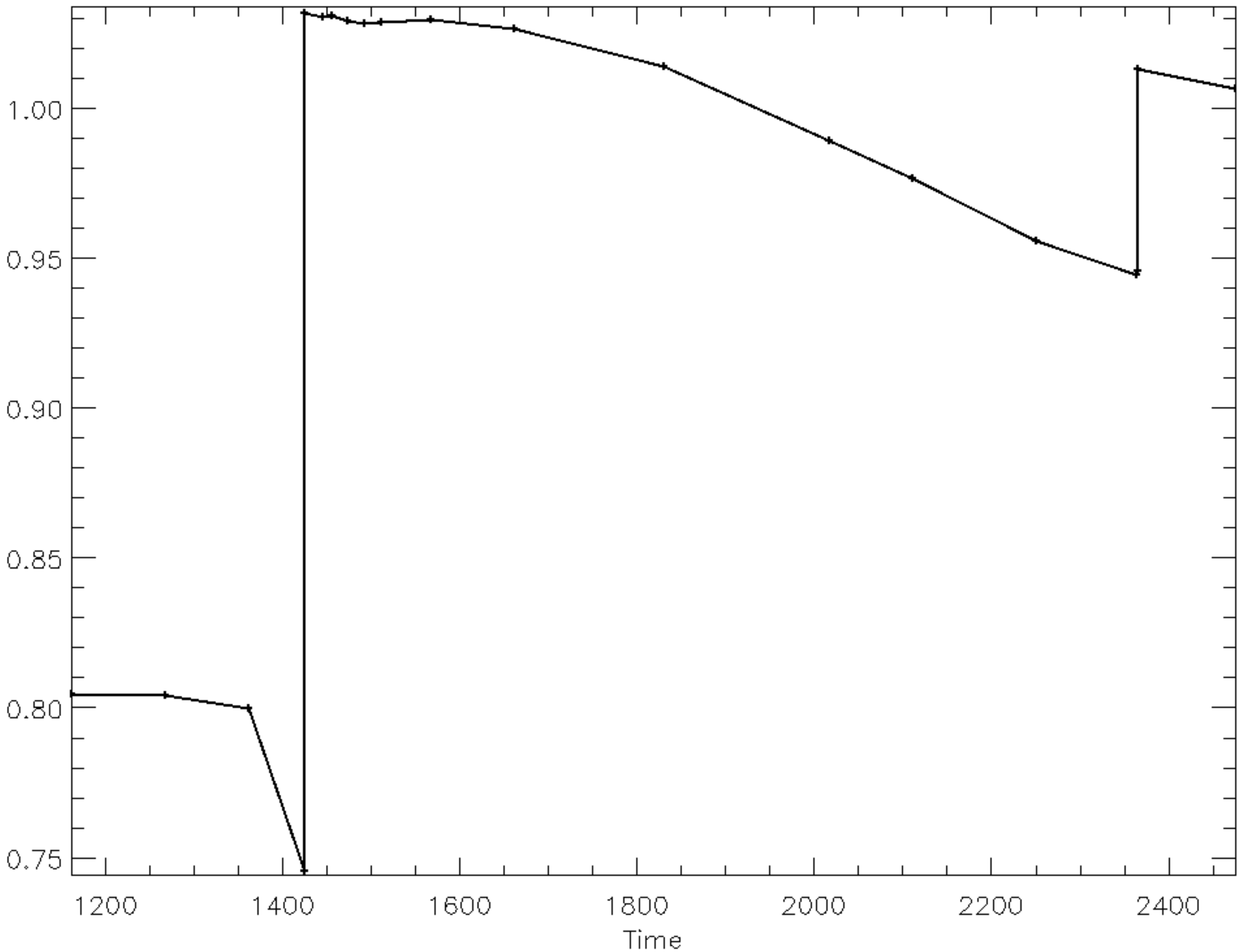
# Nonlinear fluid run: Energy history



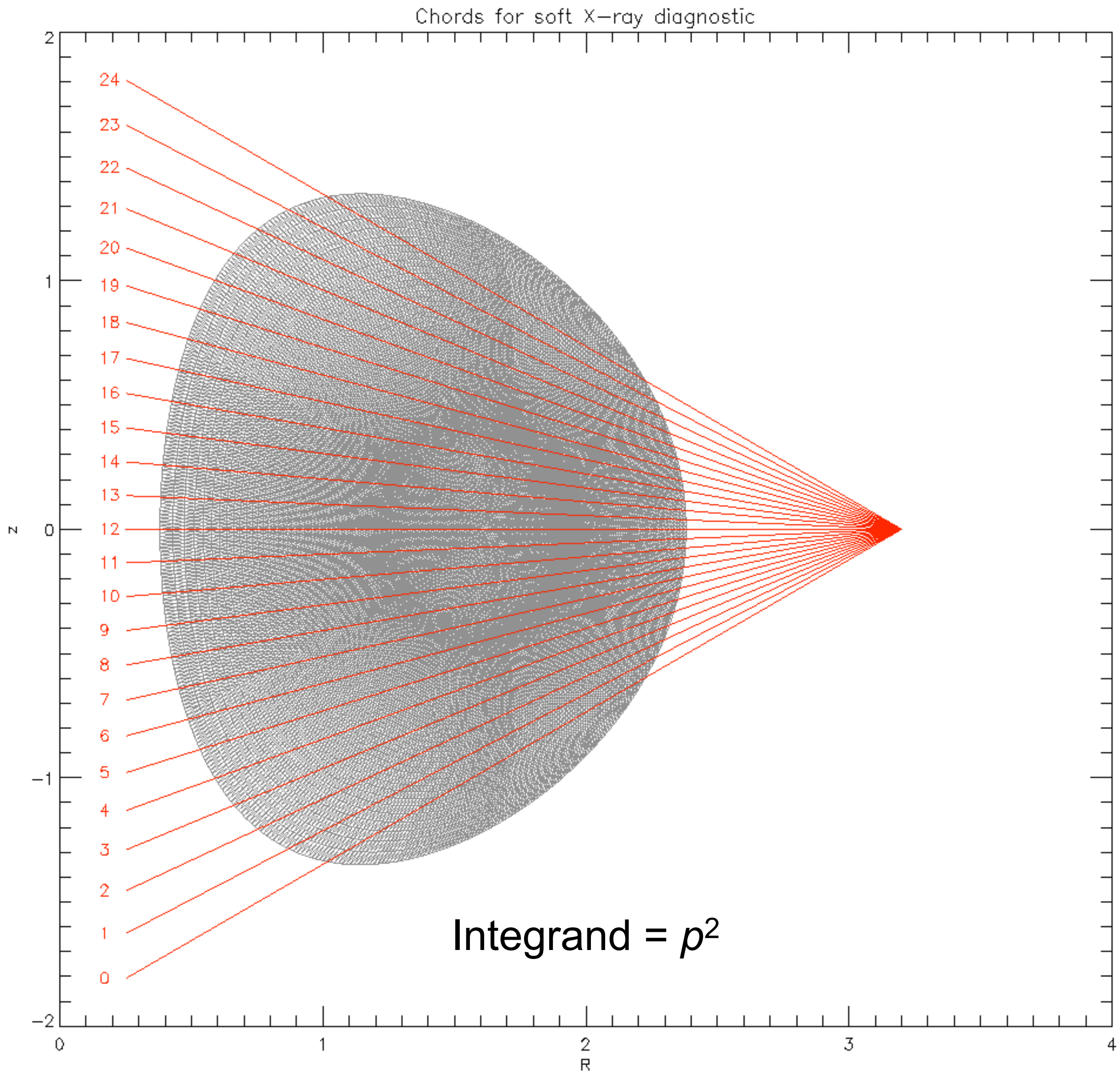
# Energy History by Mode Number



# $q_0$ history

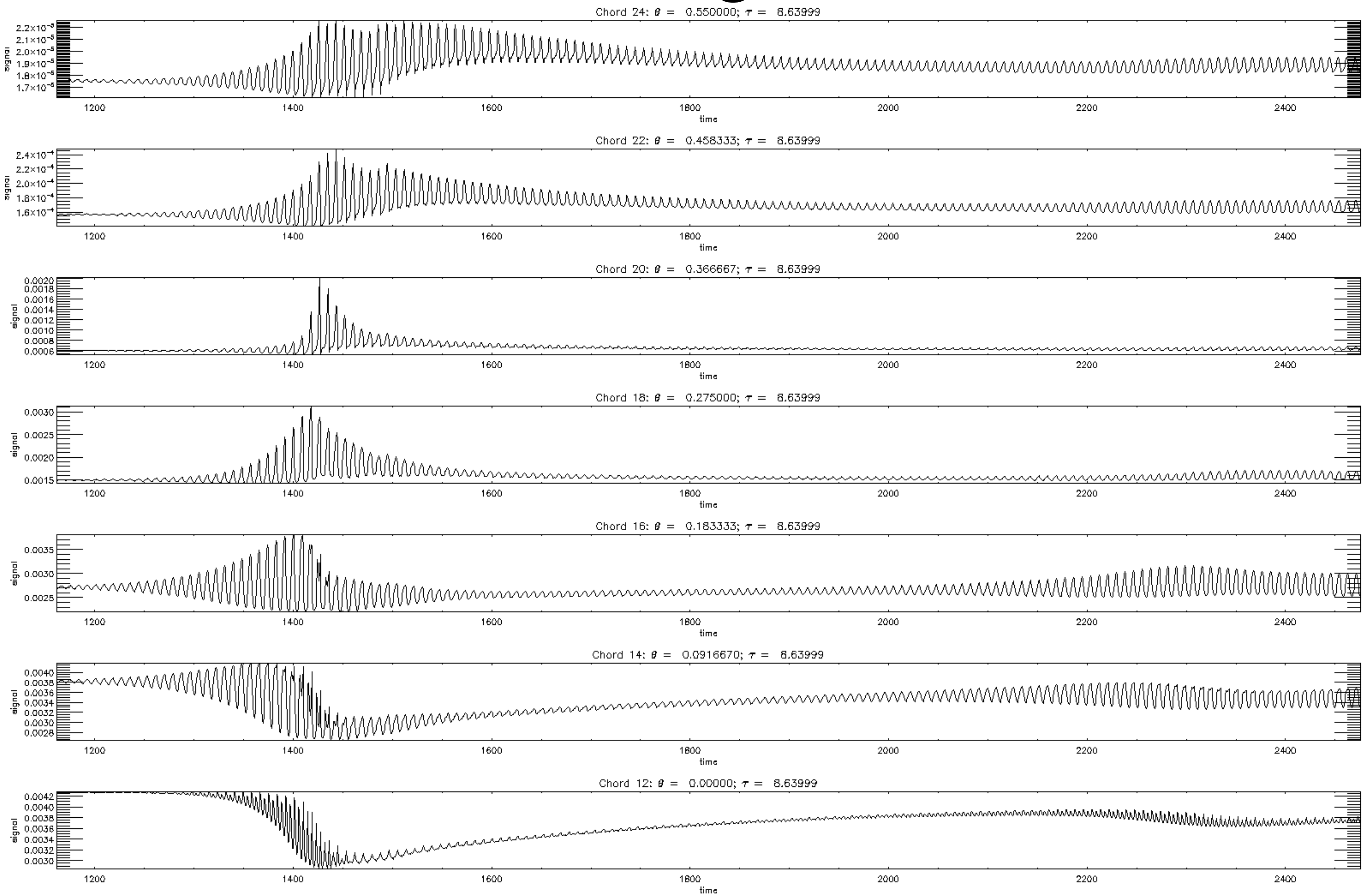

1.00
0.95
0.90
0.85
0.80
0.75
1200
1400
1600
1800
2000
2200
2400
Time

# Synthetic soft X-ray diagnostic



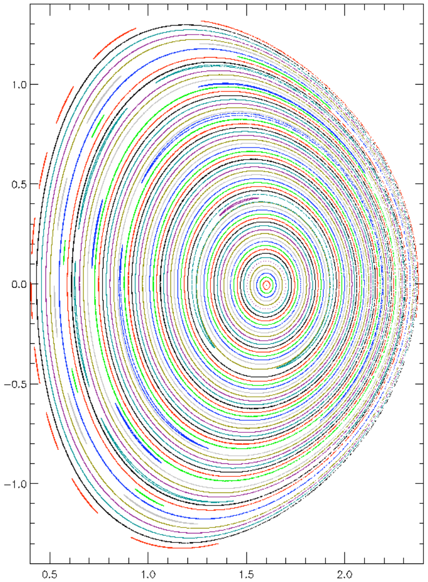

Integrand = $p^2$

# SXR signals

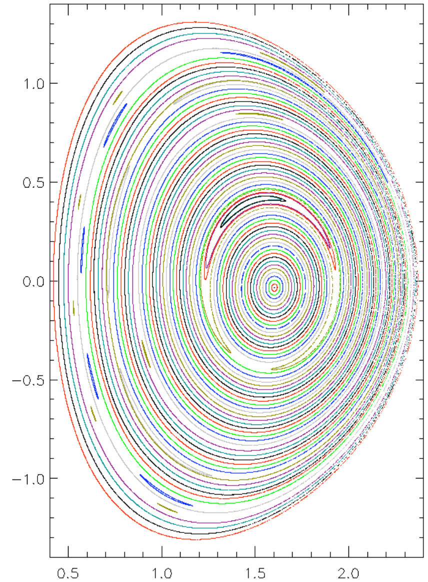


Chord 24: θ = 0.550000; τ = 8.63999
Chord 22: θ = 0.458333; τ = 8.63999
Chord 20: θ = 0.366667; τ = 8.63999
Chord 18: θ = 0.275000; τ = 8.63999
Chord 16: θ = 0.183333; τ = 8.63999
Chord 14: θ = 0.0916670; τ = 8.63999
Chord 12: θ = 0.00000; τ = 8.63999
signal
time

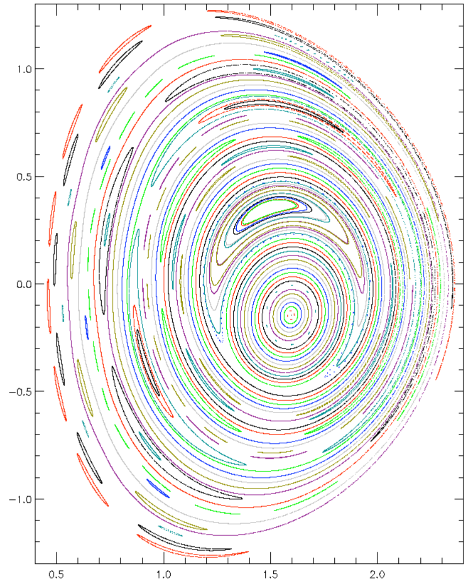
# Poincaré Plots



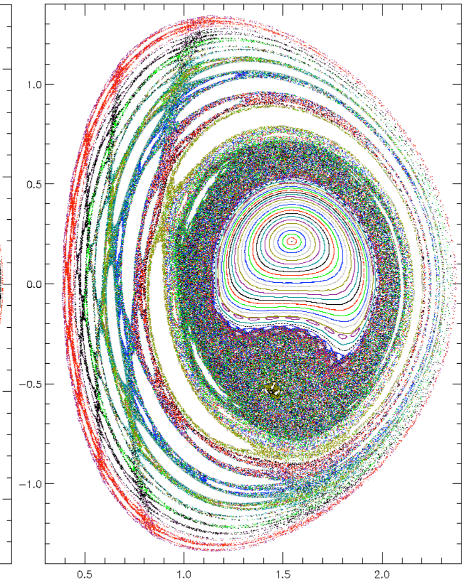
$t$ = 1161.81; $q_{min}$ = 0.8045



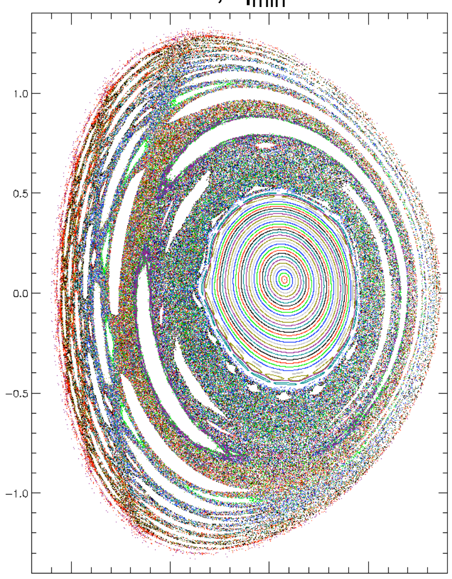
$t$ = 1268.13; $q_{min}$ = 0.8042



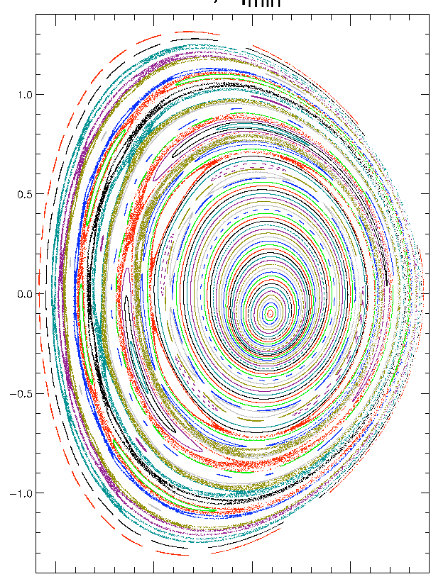
$t$ = 1361.73; $q_{min}$ = 0.7996



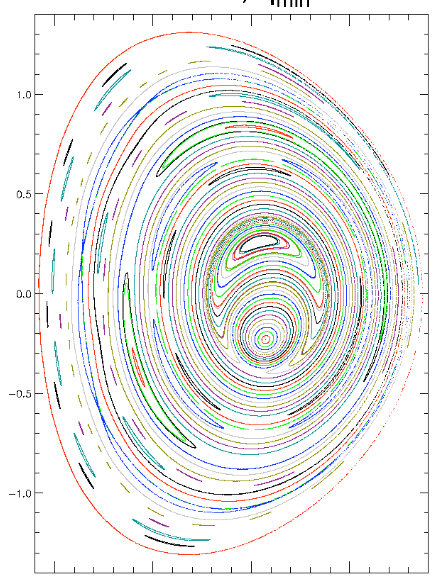
$t$ = 1425.09; $q_{min}$=0.7457



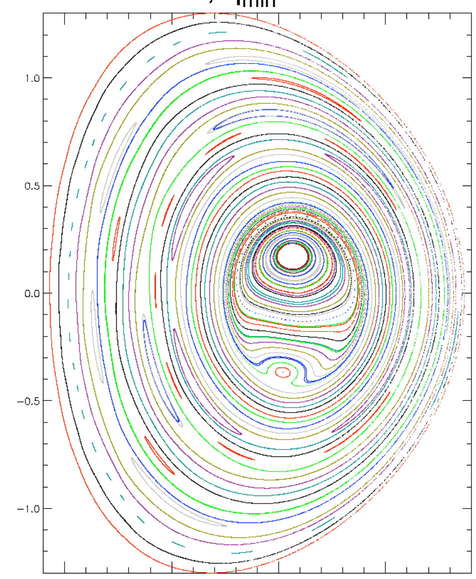
$t$ = 1445.25; $q_{min}$ = 1.0305



$t$ = 1829.73; $q_{min}$ = 1.0140

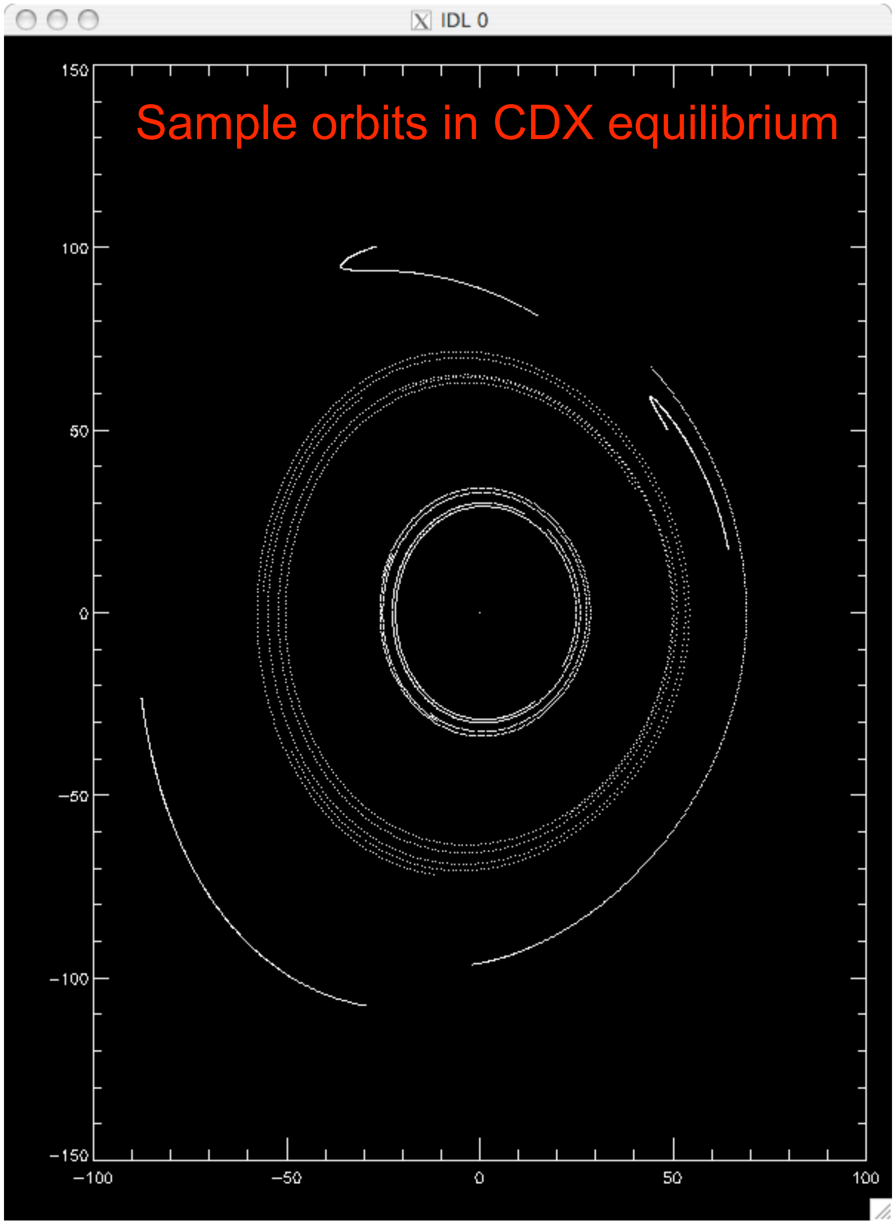


$t$ = 2250.21; $q_{min}$ = 0.9555



$t$ = 2364.69; $q_{min}$ = 0.9455

# Test particle runs have begun



# Next steps

- Adjust equilibrium to give more robust crashes with complete reconnection.
- Debug hybrid version on Franklin (currently runs on Bassi).
- Add ensemble of test particles to existing run with $\delta f$ weighting scheme (underway).
- Add ensemble of hot ions coupled by pressure tensor.
- Couple to RF code.
- Have applied for INCITE time allotment on Franklin for these studies for 2009.