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FLUID-KINETIC FORMULATION WITH EC-RF SOURCES FOR SLOW-MHD STUDIES

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I. GENERAL CONSIDERATIONS

The proposed RF/MHD approach is based on an underlying low-frequency kinetic equation, obtained after averaging over the high-frequency RF fields, of the form:

$$\frac{\partial f(\mathbf{v},\mathbf{x},t)}{\partial t} \;+\; \mathbf{v}\cdot\frac{\partial f(\mathbf{v},\mathbf{x},t)}{\partial \mathbf{x}} \;+\; \frac{e}{m}\Big(\mathbf{E}+\mathbf{v}\times\mathbf{B}\Big)\cdot\frac{\partial f(\mathbf{v},\mathbf{x},t)}{\partial \mathbf{v}} \;=\; \sum_s C(f,f_s) \;+\; Q^{RF}(f),$$

with the Fokker-Plank collision operator:

$$C(f,f_s) \;=\; -\,\frac{c^4 e^2 e_s^2 \ln\Lambda_s}{8\pi m}\,\frac{\partial}{\partial \mathbf{v}}\cdot\int d^3\mathbf{w}\;\mathsf{U}(\mathbf{v},\mathbf{w})\cdot\Big[\frac{f(\mathbf{v},\mathbf{x},t)}{m_s}\,\frac{\partial f_s(\mathbf{w},\mathbf{x},t)}{\partial \mathbf{w}} \;-\; \frac{f_s(\mathbf{w},\mathbf{x},t)}{m}\,\frac{\partial f(\mathbf{v},\mathbf{x},t)}{\partial \mathbf{v}}\Big]\,,$$

and the quasi-linear RF velocity-space diffusion operator:

$$Q^{RF}(f) \;=\; \frac{\partial}{\partial \mathbf{v}}\cdot\mathsf{D}^{RF}(\mathbf{v},\mathbf{x},t)\cdot\frac{\partial f(\mathbf{v},\mathbf{x},t)}{\partial \mathbf{v}}\;.$$

THE AVAILABLE STANDARD FORM OF THE Q^{RF} OPERATOR IS DERIVED FOR A HOMOGENEOUS BACKGROUND MAGNETIC FIELD AND VANISHING BACKGROUND ELECTRIC FIELD.

IN ORDER TO IGNORE CORRECTIONS DUE TO THE MAGNETIC INHOMOGENEITY, CONSIDER ONLY ELECTRON-CYCLOTRON RF WAVES WHOSE INTERACTION WITH THE PLASMA COULD BE TREATED IN A LOCAL APPROXIMATION.

FOR THE PROPAGATION OF THESE EC WAVES, RAY TRACING SHOULD BE SUFFICIENT.

IT IS NOT CLEAR WHAT THE CONSEQUENCES OF IGNORING THE BACKGROUND ELECTRIC FIELD IN Q^{RF} ARE.

II. ELECTRON FLUID EQUATIONS WITH RF SOURCE TERMS

Taking velocity moments of the electron kinetic equation, RF source terms appear in the generalized Ohm's law, the electron mean pressure equation and the electron pressure anisotropy (sometimes called parallel viscosity) equation:

$$\mathbf{E} \;+\; \mathbf{u}_e \times \mathbf{B} \;+\; \frac{1}{en}\Big\{\nabla \cdot \Big[p_e \mathbf{I} + (p_{e\parallel} - p_{e\perp})(\mathbf{bb} - \mathbf{I}/3)\Big] \;-\; \mathbf{F}_e^{coll} \;-\; \mathbf{F}_e^{RF}\Big\} \;=\; 0,$$

$$\frac{3}{2}\Big[\frac{\partial p_e}{\partial t} + \nabla \cdot \Big(p_e \mathbf{u}_e\Big)\Big] \;+\; p_e \nabla \cdot \mathbf{u}_e \;+\; (p_{e\parallel} - p_{e\perp})\Big\{\mathbf{b} \cdot \Big[(\mathbf{b} \cdot \nabla)\mathbf{u}_e\Big] - \nabla \cdot \mathbf{u}_e/3\Big\} +$$

$$+ \nabla \cdot \Big(q_{e\parallel}\mathbf{b} + \mathbf{q}_{e\perp}\Big) \;-\; g_e^{coll} \;-\; g_e^{RF} \;=\; 0 \;,$$

$$\frac{\partial (p_{e\parallel} - p_{e\perp})}{\partial t} \;+\; \nabla \cdot \Big[(p_{e\parallel} - p_{e\perp})\mathbf{u}_e\Big] \;+\; (p_{e\parallel} - p_{e\perp})\Big\{\mathbf{b} \cdot \Big[(\mathbf{b} \cdot \nabla)\mathbf{u}_e\Big] + \nabla \cdot \mathbf{u}_e/3\Big\} +$$

$$+ p_e\Big\{\mathbf{b} \cdot \Big[3(\mathbf{b} \cdot \nabla)\mathbf{u}_e\Big] - \nabla \cdot \mathbf{u}_e\Big\} \;+\; \nabla \cdot \Big[(3q_{eB\parallel} - q_{e\parallel})\mathbf{b} + 3\mathbf{q}_{eB\perp} - \mathbf{q}_{e\perp}\Big] +$$

$$+ 3(q_{e\parallel} - q_{eB\parallel})\mathbf{b} \cdot \nabla(\ln\; B) \;-\; 6\mathbf{q}_{eB\perp} \cdot \boldsymbol{\kappa} \;+\; g_e^{coll} \;+\; g_e^{RF} \;-\; 3g_{eB}^{coll} \;-\; 3g_{eB}^{RF} \;=\; 0 \;.$$

The RF source terms in the electron equations are:

$$\mathbf{F}_e^{RF} = m_e \int d^3\mathbf{v}\ (\mathbf{v} - \mathbf{u}_e)\ Q^{RF}(f_e),$$

$$g_e^{RF} = (m_e/2) \int d^3\mathbf{v}\ |\mathbf{v} - \mathbf{u}_e|^2\ Q^{RF}(f_e),$$

$$g_{eB}^{RF} = (m_e/2) \int d^3\mathbf{v}\ [(\mathbf{v} - \mathbf{u}_e) \cdot \mathbf{b}]^2\ Q^{RF}(f_e)$$

For the slow instabilities of interest, the electron distribution function can be assumed to be close to a Maxwellian. Since the amplitude of the RF source terms is also small, these can be approximated by the corresponding explicit representations obtained with a Maxwellian distribution $f_e = f_{eM}$**.**

In the collisionality regimes of interest, the collisional (diffusive) parts of the perpendicular heat fluxes should be negligible. The corresponding collision-independent (diamagnetic) parts are also completely specified within the required accuracy if the lowest-order electron distribution function is (two-temperature) Maxwellian:

$$\mathbf{q}_{e\perp} \;=\; -\frac{1}{eB}\mathbf{b}\times\Big[p_{e\perp}\nabla\Big(\frac{p_{e\parallel}+4p_{e\perp}}{2n}\Big)+\frac{p_{e\parallel}(p_{e\parallel}-p_{e\perp})}{n}\boldsymbol{\kappa}\Big]\;,$$

$$\mathbf{q}_{eB\perp} \;=\; -\frac{1}{eB}\mathbf{b}\times\Big[p_{e\perp}\nabla\Big(\frac{p_{e\parallel}}{2n}\Big)+\frac{p_{e\parallel}(p_{e\parallel}-p_{e\perp})}{n}\boldsymbol{\kappa}\Big]\;.$$

The electron closure terms that must be provided by kinetic theory are:

The two independent parallel heat fluxes:

$$q_{e\|} \;=\; (m_e/2)\int d^3\mathbf{v}\;[(\mathbf{v}-\mathbf{u}_e)\cdot\mathbf{b}]\;|\mathbf{v}-\mathbf{u}_e|^2\;f_e\;,$$

$$q_{eB\|} \;=\; (m_e/2)\int d^3\mathbf{v}\;[(\mathbf{v}-\mathbf{u}_e)\cdot\mathbf{b}]^3\;f_e\;.$$

The collisional moments:

$$\mathbf{F}_e^{coll} \;=\; m_e\int d^3\mathbf{v}\;(\mathbf{v}-\mathbf{u}_e)\;\sum_s C(f_e,f_s),$$

$$g_e^{coll} \;=\; (m_e/2)\int d^3\mathbf{v}\;|\mathbf{v}-\mathbf{u}_e|^2\;\sum_s C(f_e,f_s),$$

$$g_{eB}^{coll} \;=\; (m_e/2)\int d^3\mathbf{v}\;[(\mathbf{v}-\mathbf{u}_e)\cdot\mathbf{b}]^2\;\sum_s C(f_e,f_s)$$

III. DRIFT-KINETIC EVALUATION OF THE FLUID CLOSURES

The electron fluid closure terms can be evaluated from the solution of a gyrophase-averaged drift-kinetic equation. Such equation should fulfill the following requirements:

Three spatial dimensions in order to analyze non-axisymetric, non-linear magnetic island evolution.

Two velocity dimensions. Magnetic moment no longer conserved when RF and FLR effects are included.

Velocity moments in agreement with the macroscopic fluid equations. Include terms beyond the lowest (zero-Larmor-radius) order, that are inversely proportional to eB but independent of the mass.

A FINITE-LARMOR-RADIUS FORM OF THE DRIFT-KINETIC EQUATION HAS BEEN DERIVED, HAVING THE FOLLOWING DESIRABLE FEATURES:

Accurate to the first significant finite-Larmor-radius order and valid for arbitrary macroscopic flows.

Use of the full macroscopic flow velocity, u, to define the moving frame. Electric field exactly eliminated algebraically and no reference to the $\mathbf{E} \times \mathbf{B}$ or any other drifts.

Formulation in terms of the standard MHD variables (macroscopic flow velocity and magnetic field) only. This should facilitate the coupling to M3D, NIMROD or other MHD-like codes.

Velocity moments reproduce all the previously derived fluid results, including the higher-moment and higher-order in the Larmor radius results.

In its full generality, this "driftless" FLR drift-kinetic equation is (for any species and dropping the species index):

$$\frac{\partial \bar{f}(v'_\parallel, v'_\perp, \mathbf{x}, t)}{\partial t} \; + \; \dot{\mathbf{x}} \cdot \frac{\partial \bar{f}}{\partial \mathbf{x}} \; + \; \dot{v}'_\parallel \, \frac{\partial \bar{f}}{\partial v'_\parallel} \; + \; \dot{v}'_\perp \, \frac{\partial \bar{f}}{\partial v'_\perp} \; = \; \bar{C} \; + \; \bar{Q}^{RF}(f_M) \; ,$$

where the relative velocity variable is $\mathbf{v}' = \mathbf{v} - \mathbf{u}(\mathbf{x}, t)$, with $\mathbf{u}(\mathbf{x}, t)$ the macroscopic flow velocity, and the overbars indicate gyrophase averaging.

The coefficient functions are:

$$\dot{\mathbf{x}} \; = \; \mathbf{u} + v'_\parallel \mathbf{b} \; + \; \frac{v'^2_\perp}{2} \nabla \times \frac{\mathbf{b}}{\Omega_c} \; + \; \frac{\mathbf{b}}{\Omega_c} \times \Big[\frac{\mathbf{F}}{mn} + 2 v'_\parallel (\mathbf{b} \cdot \nabla) \mathbf{u} + \Big(v'^2_\parallel - \frac{v'^2_\perp}{2} \Big) \boldsymbol{\kappa} \Big]$$

with

$$\mathbf{F}(\mathbf{x}, t) \; = \; - \, en(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \; + \; mn \Big[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \Big] \; = \; - \, \nabla \cdot \mathsf{P} \; + \; \mathbf{F}^{coll} \; ,$$

$$\dot{v}'_{\parallel} \;=\; -\,\frac{\mathbf{b}\cdot\mathbf{F}}{mn} \;-\; v'_{\parallel}\mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)\mathbf{u}] \;-\; \frac{v'^2_{\perp}}{2}\mathbf{b}\cdot\nabla\ln B \;+\; \frac{v'^2_{\perp}}{2}\nabla\cdot\Big[\frac{\mathbf{b}}{\Omega_c}\times(\boldsymbol{\omega}\times\mathbf{b}+v'_{\parallel}\boldsymbol{\kappa})\Big] \;-$$

$$-\,\Big[\frac{\mathbf{b}}{\Omega_c}\times(\boldsymbol{\omega}\times\mathbf{b}+v'_{\parallel}\boldsymbol{\kappa})\Big]\cdot\Big[\frac{\mathbf{F}}{mn}+2v'_{\parallel}(\mathbf{b}\cdot\nabla)\mathbf{u}+v'^2_{\parallel}\boldsymbol{\kappa}\Big] \;-\; \frac{v'^2_{\perp}}{2}\sigma$$

with

$$\boldsymbol{\omega}(\mathbf{x},t) \;=\; \nabla\times\mathbf{u} \qquad \textit{and} \qquad \sigma(\mathbf{x},t) \;=\; \frac{1}{4\Omega_c}\epsilon_{jkl}b_j\Big(\frac{\partial b_k}{\partial x_m}+\frac{\partial b_m}{\partial x_k}\Big)(\delta_{mn}-b_m b_n)\Big(\frac{\partial u_l}{\partial x_n}+\frac{\partial u_n}{\partial x_l}\Big)\;,$$

and

$$\dot{v}'_{\perp} \;=\; \frac{v'_{\perp}}{2}\Big\{\mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)\mathbf{u}] \;-\nabla\cdot\mathbf{u} \;+\; v'_{\parallel}\mathbf{b}\cdot\nabla\ln B \;-\; \nabla\cdot\Big[\frac{\mathbf{b}}{\Omega_c}\times\Big(\frac{\mathbf{F}}{mn}+2v'_{\parallel}(\mathbf{b}\cdot\nabla)\mathbf{u}+v'^2_{\parallel}\boldsymbol{\kappa}\Big)\Big] \;+$$

$$+\;2\Big[\frac{\mathbf{b}}{\Omega_c}\times(\boldsymbol{\omega}\times\mathbf{b})\Big]\cdot\Big[(\mathbf{b}\cdot\nabla)\mathbf{u}+v'_{\parallel}\boldsymbol{\kappa}\Big] \;+\; \Big(\frac{\mathbf{b}}{\Omega_c}\times\boldsymbol{\kappa}\Big)\cdot\Big[\frac{\mathbf{F}}{mn}+4v'_{\parallel}(\mathbf{b}\cdot\nabla)\mathbf{u}\Big]\Big\}\;.$$

FOR SMALL-MASS ELECTRONS, SLOW TIME EVOLUTION ($\partial/\partial t < \omega_*$), SLOW FLOWS ($u < u_*$), AND USING AS PHASE-SPACE VARIABLES THE KINETIC ENERGY $\varepsilon = m(v_\parallel'^2 + v_\perp'^2)/2$ AND THE MAGNETIC MOMENT $\mu = mv_\perp'^2/(2B)$:

$$\dot{\mathbf{x}} \cdot \frac{\partial \bar{f}(\varepsilon, \mu, \mathbf{x}, t)}{\partial \mathbf{x}} \;+\; \dot{\varepsilon}\, \frac{\partial \bar{f}}{\partial \varepsilon} \;+\; \dot{\mu}\, \frac{\partial \bar{f}}{\partial \mu} \;=\; \bar{C} \;+\; \bar{Q}^{RF}(f_M) \;,$$

where

$$\dot{\mathbf{x}} \;=\; \Big[2(\varepsilon - \mu B)/m\Big]^{1/2} \mathbf{b} \;+\; \mu B\, \nabla \times \frac{\mathbf{b}}{m\Omega_c} \;+\; \frac{\mathbf{b}}{m\Omega_c} \times \Big[\frac{\mathbf{F}}{n} + (2\varepsilon - 3\mu B)\boldsymbol{\kappa}\Big] \;,$$

$$\dot{\varepsilon} \;=\; -\Big[2(\varepsilon - \mu B)/m\Big]^{1/2} \frac{\mathbf{b}\cdot\mathbf{F}}{n} \;-\; \mu B\, \nabla \cdot \Big(\frac{\mathbf{b}}{m\Omega_c} \times \frac{\mathbf{F}}{n}\Big) \;+\; (2\varepsilon - 3\mu B)\Big(\frac{\mathbf{b}}{m\Omega_c} \times \frac{\mathbf{F}}{n}\Big) \cdot \boldsymbol{\kappa} \;,$$

$$\dot{\mu} \;=\; \frac{\mu}{m\Omega_c} \mathbf{b} \cdot \Big\{ \nabla \times \Big[2(\varepsilon - \mu B)\boldsymbol{\kappa}\Big] \;-\; \left[\mathbf{b} \cdot (\nabla \times \mathbf{b})\right] \Big(\frac{\mathbf{F}}{n} + \mu \nabla B\Big) \Big\} \;.$$

IV. ACTION ITEM

IN ORDER TO PROCEED IN A COHERENT FASHION, IT WOULD BE HIGHLY DESIRABLE TO REACH A CONSENSUS ON THE APPROPRIATE RELATIVE ORDERINGS AMONG THE FOLLOWING DIMENSIONLESS RATIOS:

$$m_e/m_i \ \ , \ \ \rho_i/L \ \ , \ \ (\partial/\partial t)/\Omega_{ci} \ \ , \ \ \nu_i/\Omega_{ci} \ \ , \ \ u_s/v_{thi} \ \ , \ \ (p_{s\parallel} - p_{s\perp})/p_s \ \ , \ \ (f_s - f_{sM})/f_{sM}$$