NIMROD simulations with simple current sources

Tom Jenkins

University of Wisconsin-Madison

in collaboration with

Dalton Schnack, Carl Sovinec, Chris Hegna, Jim Callen, Fatima Ebrahimi
University of Wisconsin-Madison

Scott Kruger, Johan Carlsson
Tech-X Corporation

Eric Held, Jeong-Young Ji
Utah State University

About NIMROD

- Evolves fluid equations for magnetically confined plasmas — stiff, anisotropic system
- Finite elements in poloidal plane; Fourier decomposition in toroidal direction
- Can be tailored to many plasma configurations: tokamak, spheromak, astrophysical, etc.

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} + \kappa_b \nabla(\nabla \cdot \mathbf{B}) \qquad \mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = \nabla \cdot (D \nabla n) \qquad \mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \mathbf{J} \times \mathbf{B} + \nabla(\nu \rho \nabla \mathbf{v})$$

$$\frac{n}{\gamma - 1}\left(\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla)T\right) = -\frac{p}{2}\nabla \cdot \mathbf{v} - \nabla \cdot q + Q$$

- Numerical dissipation introduced

Insert an ad hoc term (general RF effects) in Ohm's law

- MHD: Ohm's law:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

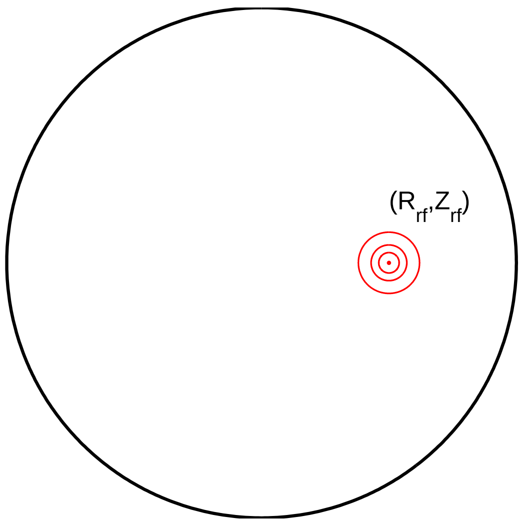
- Determine effect of current drive (not self-consistent): ad hoc source term

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta (\mathbf{J} - \mathbf{J}_{\mathrm{rf}})$$

- Should modify plasma equilibrium. Assume axisymmetric current profile of form

$$\mathbf{J}_{\mathrm{rf}}(R, Z, t) = \lambda_{\mathrm{rf}} \exp\left[\frac{-\left[(R - R_{\mathrm{rf}})^2 + (Z - Z_{\mathrm{rf}})^2\right]}{w_{\mathrm{rf}}^2}\right] \frac{\mathbf{B}}{\mu_0} f(t)$$

- Can also consider this as $\mathbf{E} \rightarrow \mathbf{E} + \eta \mathbf{J}_{\mathrm{rf}}$, where $\eta \mathbf{J}_{\mathrm{rf}}$ is an emf-per-unit-length inducing magnetic field in the plasma.



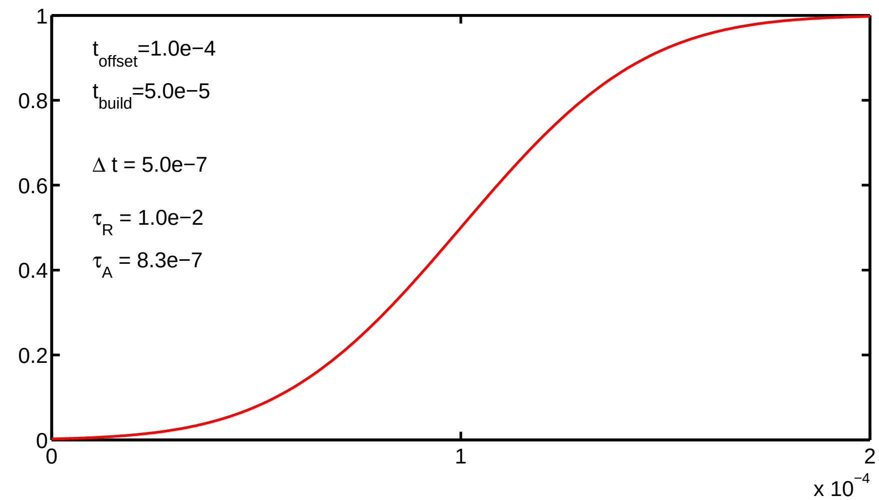
- Variable parameters $\lambda_{\mathrm{rf}}, R_{\mathrm{rf}}, Z_{\mathrm{rf}}, w_{\mathrm{rf}}$ (amplitude, location, spatial width), along with time dependence $f(t)$; begin with cylindrical equilibrium.
- Source *not* averaged over flux surface

Ad hoc RF current can be slowly ramped up in time

- Specify the time dependence $f(t)$:

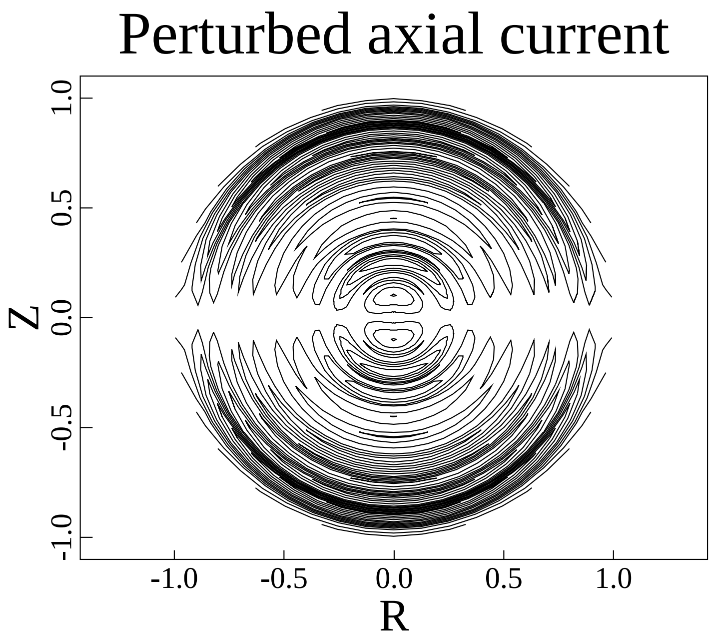
$$f(t) = \left[\frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{t - t_{\mathrm{offset}}}{t_{\mathrm{build}}}\right)\right]$$

- Build up current on timescale t_{build} to suppress transient Alfvén waves (t_{build} short compared to resistive diffusion timescale τ_R)

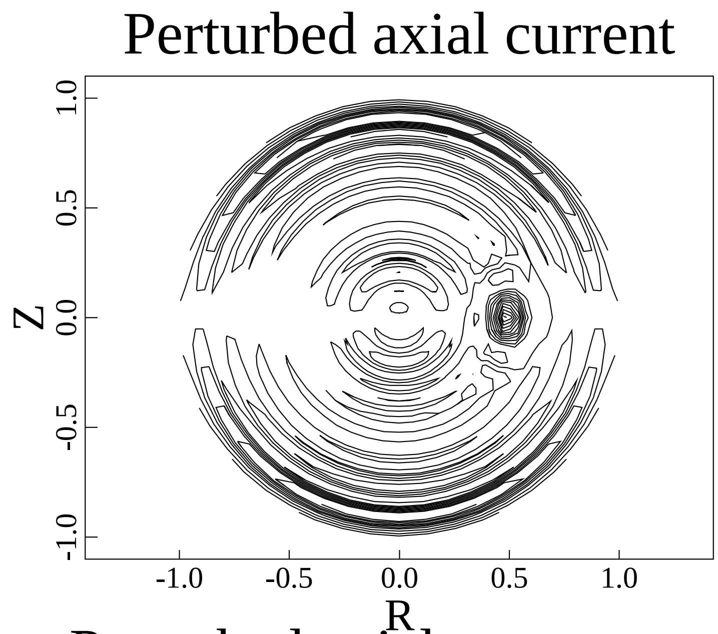


- Initial perturbation has amplitude comparable to random current fluctuations; $\lambda_{\mathrm{rf}} = 8.0 \times 10^{-4}$, $w_{\mathrm{rf}} = 0.1$, $R_{\mathrm{rf}} = 0.5$, $Z_{\mathrm{rf}} = 0$

Current equilibrates over a flux surface in a few Alfvén times

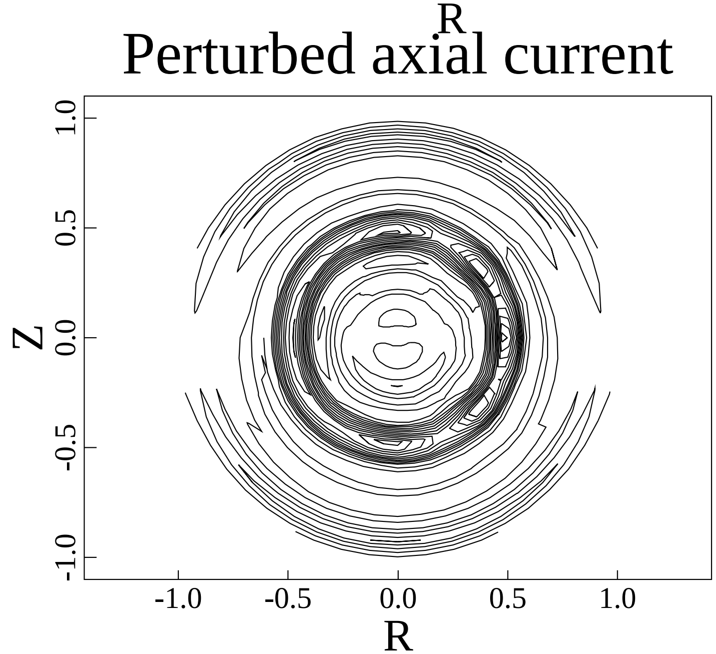


$t = 0$



$t = \Delta t$

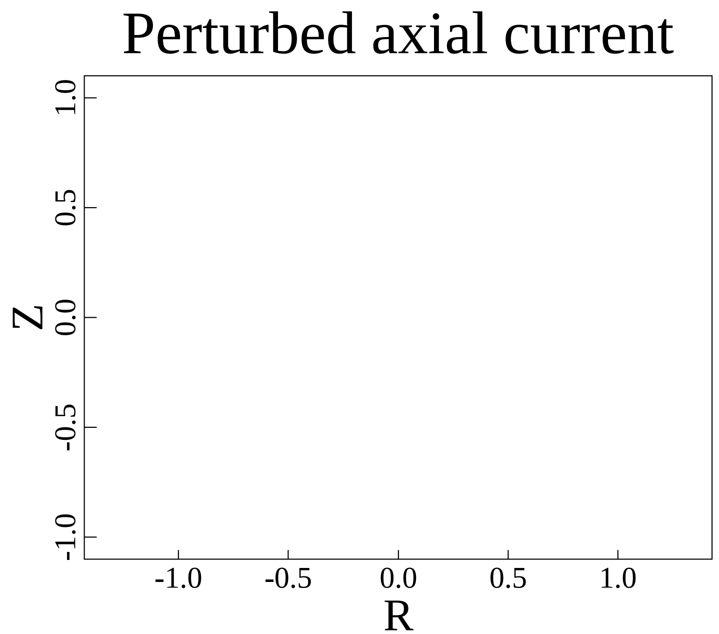
- $\Delta t \approx 0.6\tau_A$, after only a few Alfvén times, current is spread across entire flux surface
- Initial condition induces small–amplitude Alfvén waves (not related to RF source)



$t = 20\Delta t$

Current equilibration: Alfvén timescale

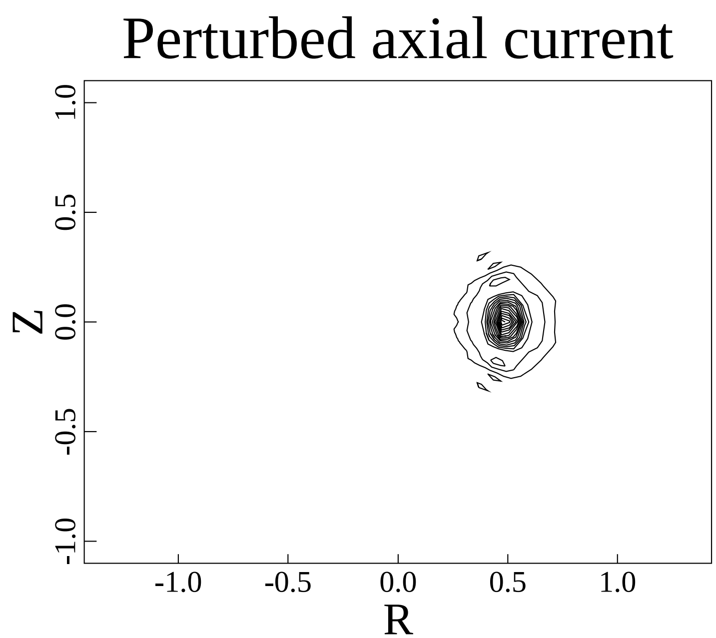
- Examine this process more closely by reducing the timestep and increasing λ_{rf}
- $\lambda_{\mathrm{rf}} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 0$

Current equilibration: Alfvén timescale

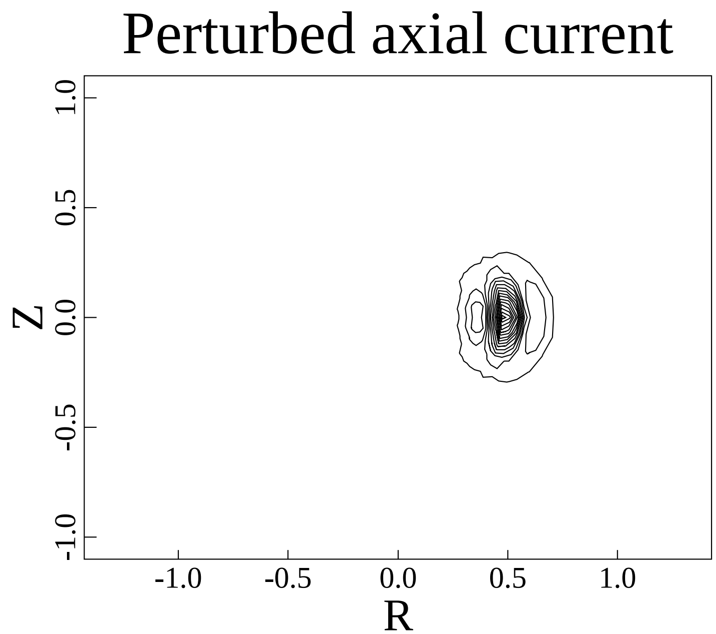
- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 0.10 \times 10^{-6}$

Current equilibration: Alfvén timescale

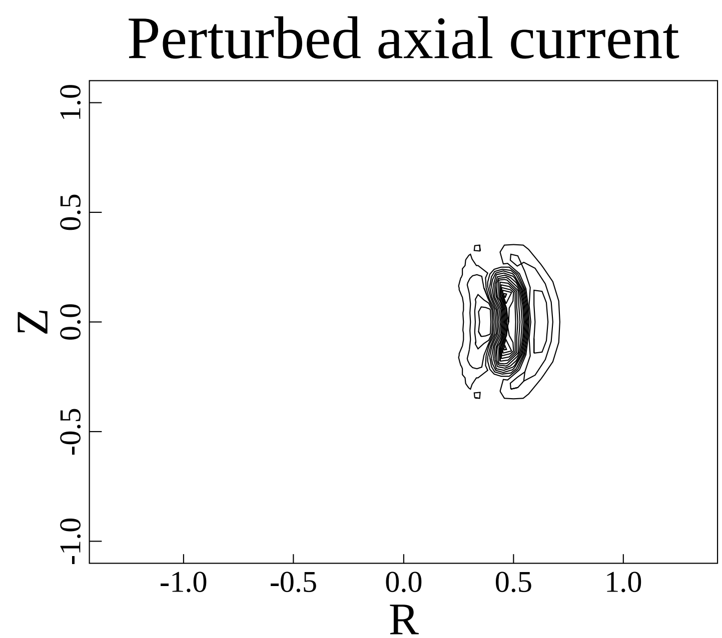
- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 0.20 \times 10^{-6}$

Current equilibration: Alfvén timescale

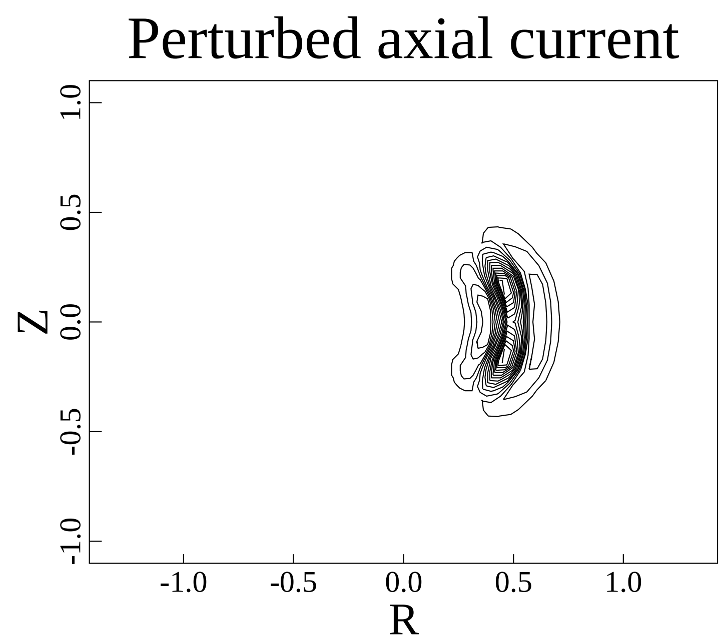
- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 0.30 \times 10^{-6}$

Current equilibration: Alfvén timescale

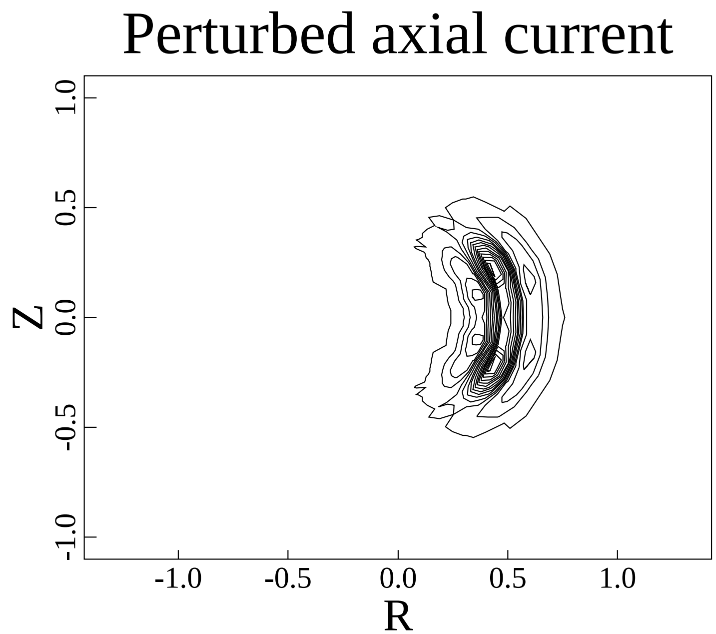
- Examine this process more closely by reducing the timestep and increasing λ_{rf}
- $\lambda_{\mathrm{rf}} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 0.40 \times 10^{-6}$

Current equilibration: Alfvén timescale

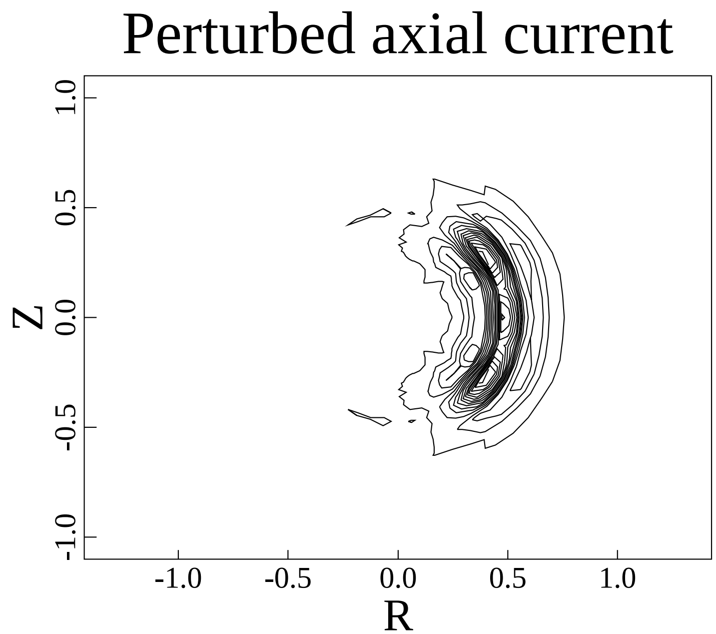
- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 0.50 \times 10^{-6}$

Current equilibration: Alfvén timescale

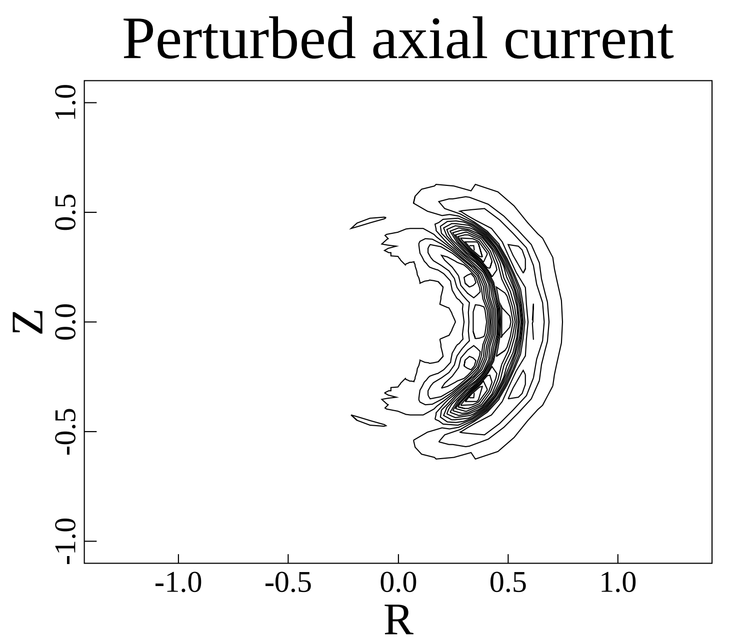
- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 0.60 \times 10^{-6}$

Current equilibration: Alfvén timescale

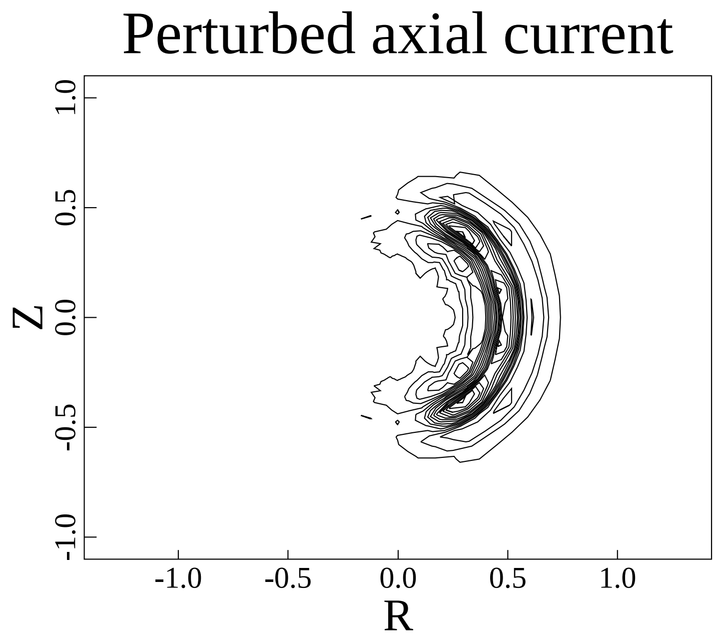
- Examine this process more closely by reducing the timestep and increasing λ_{rf}
- $\lambda_{\mathrm{rf}} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 0.70 \times 10^{-6}$

Current equilibration: Alfvén timescale

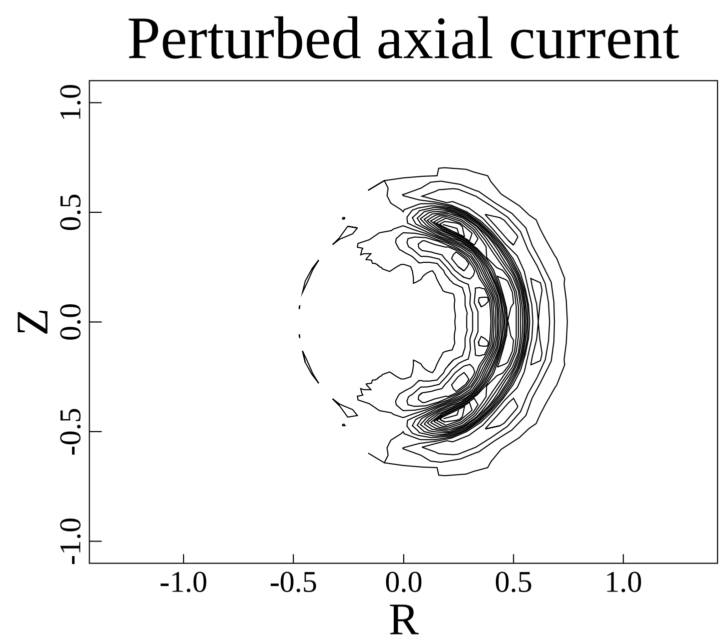
- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 0.80 \times 10^{-6}$

Current equilibration: Alfvén timescale

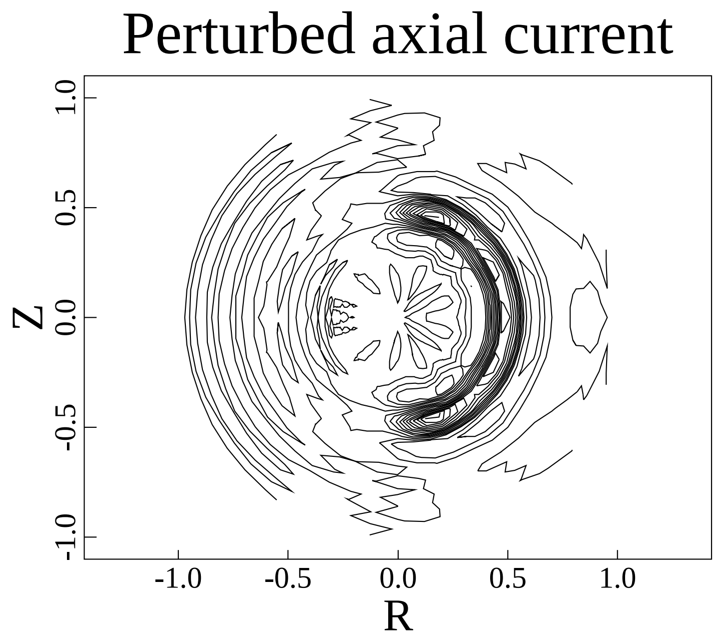
- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 0.90 \times 10^{-6}$

Current equilibration: Alfvén timescale

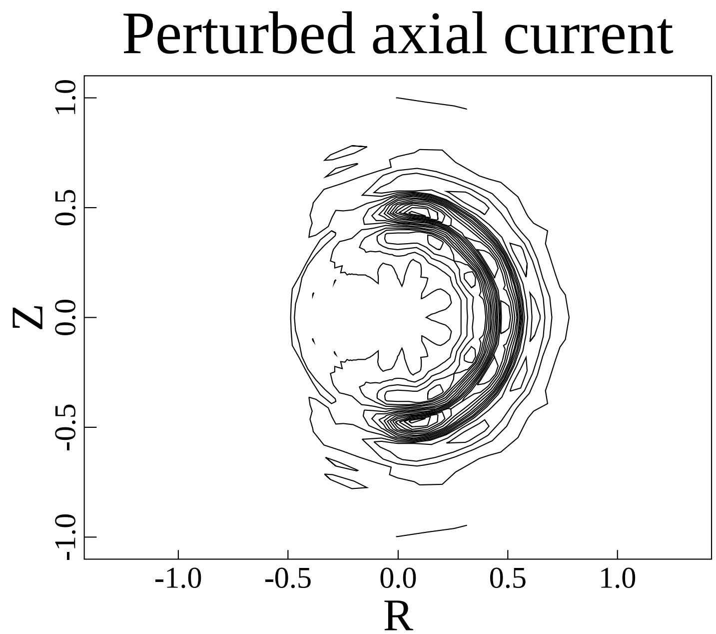
- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 1.00 \times 10^{-6}$

Current equilibration: Alfvén timescale

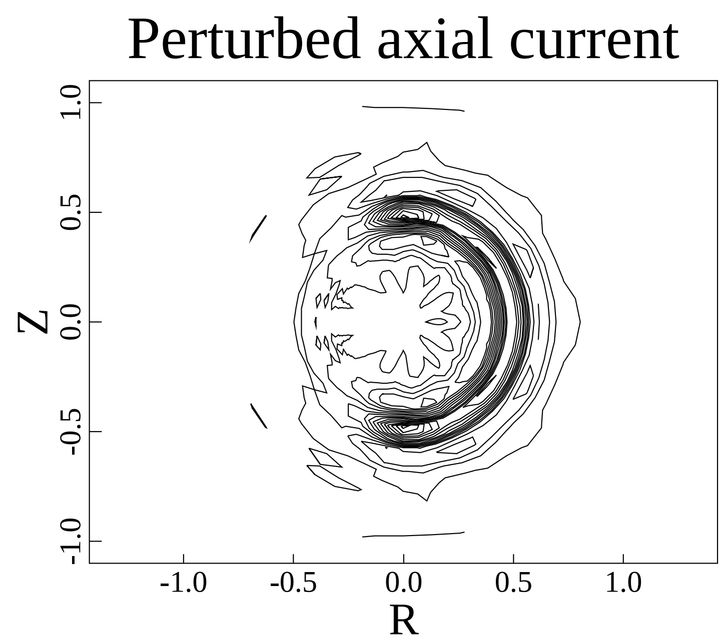
- Examine this process more closely by reducing the timestep and increasing λ_{rf}
- $\lambda_{\mathrm{rf}} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 1.10 \times 10^{-6}$

Current equilibration: Alfvén timescale

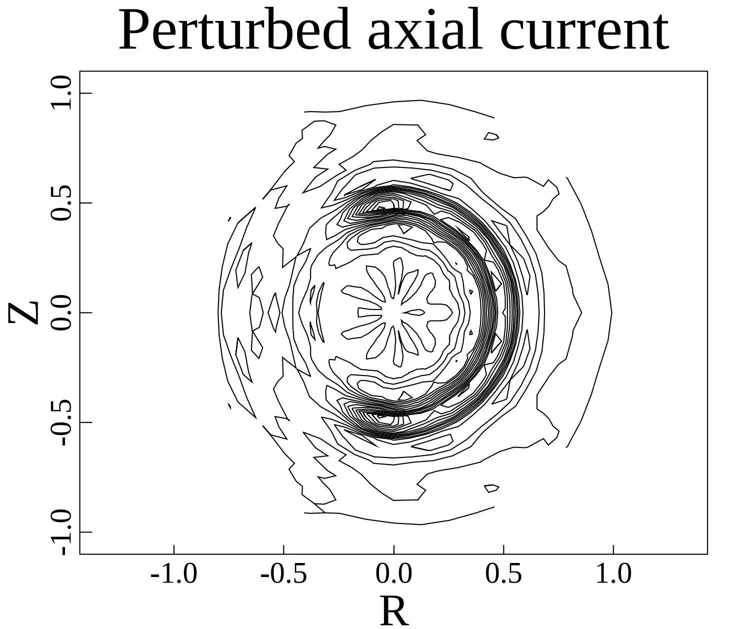
- Examine this process more closely by reducing the timestep and increasing λ_{rf}
- $\lambda_{\mathrm{rf}} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 1.20 \times 10^{-6}$

Current equilibration: Alfvén timescale

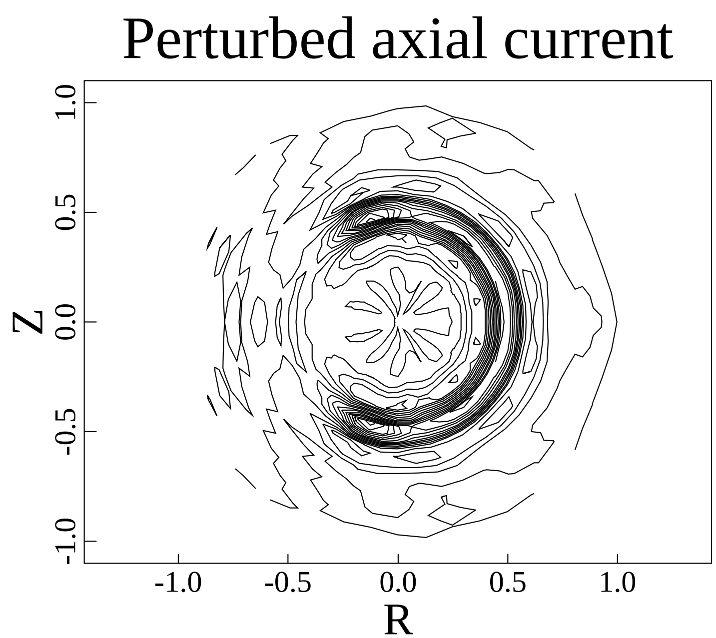
- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 1.30 \times 10^{-6}$

Current equilibration: Alfvén timescale

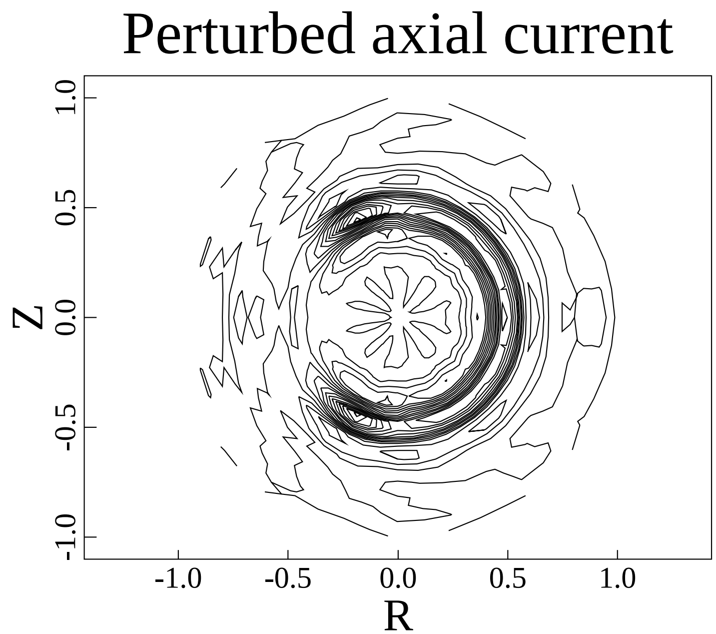
- Examine this process more closely by reducing the timestep and increasing λ_{rf}
- $\lambda_{\mathrm{rf}} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 1.40 \times 10^{-6}$

Current equilibration: Alfvén timescale

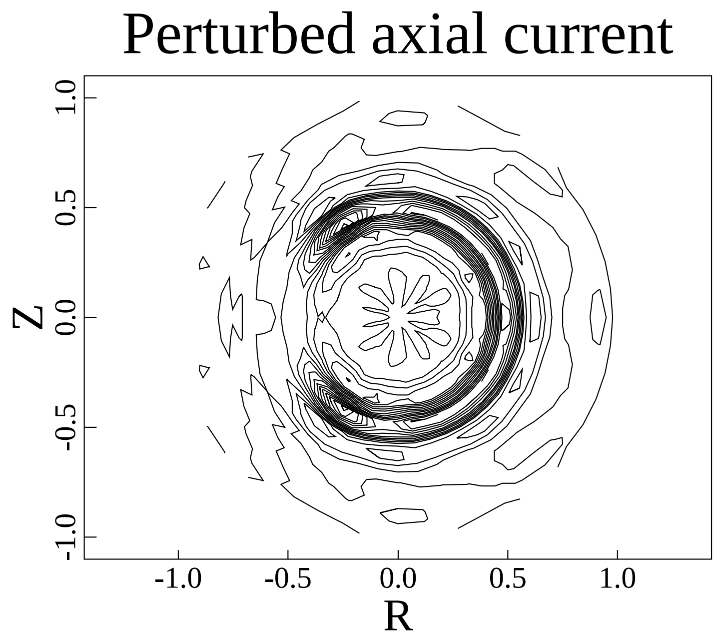
- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 1.50 \times 10^{-6}$

Current equilibration: Alfvén timescale

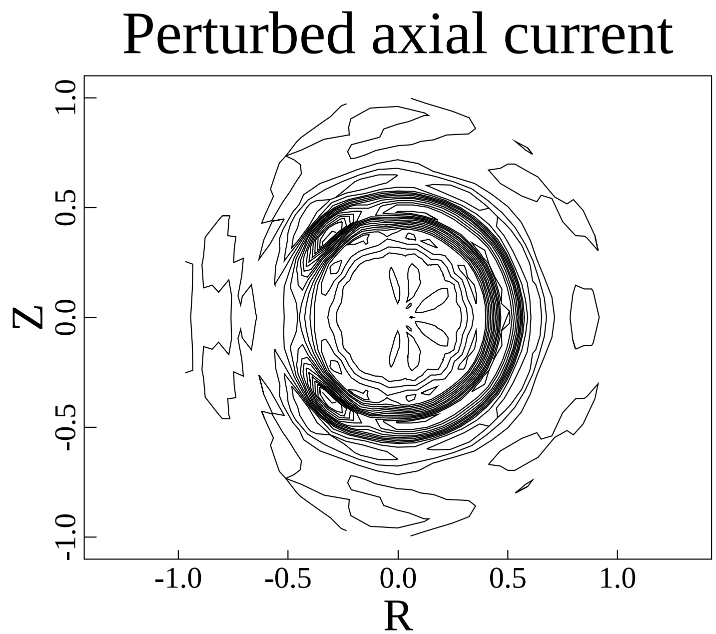
- Examine this process more closely by reducing the timestep and increasing λ_{rf}
- $\lambda_{\mathrm{rf}} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 1.60 \times 10^{-6}$

Current equilibration: Alfvén timescale

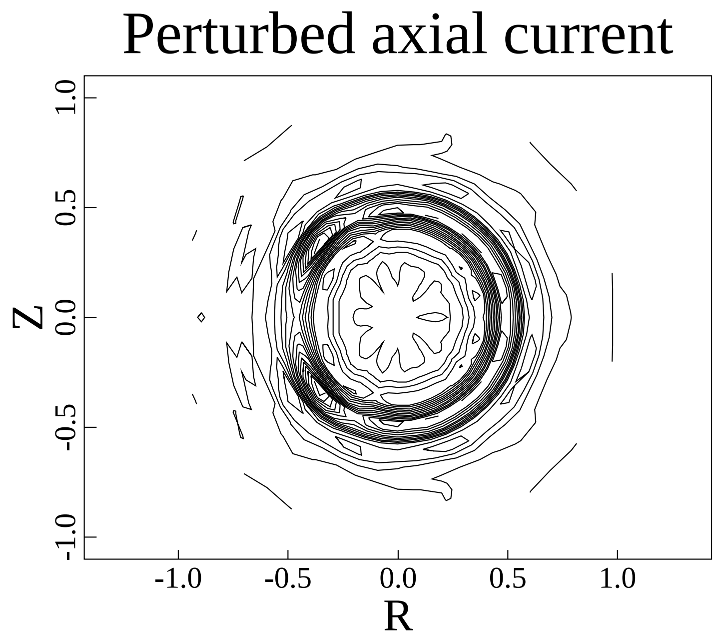
- Examine this process more closely by reducing the timestep and increasing λ_{rf}
- $\lambda_{\mathrm{rf}} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 1.70 \times 10^{-6}$

Current equilibration: Alfvén timescale

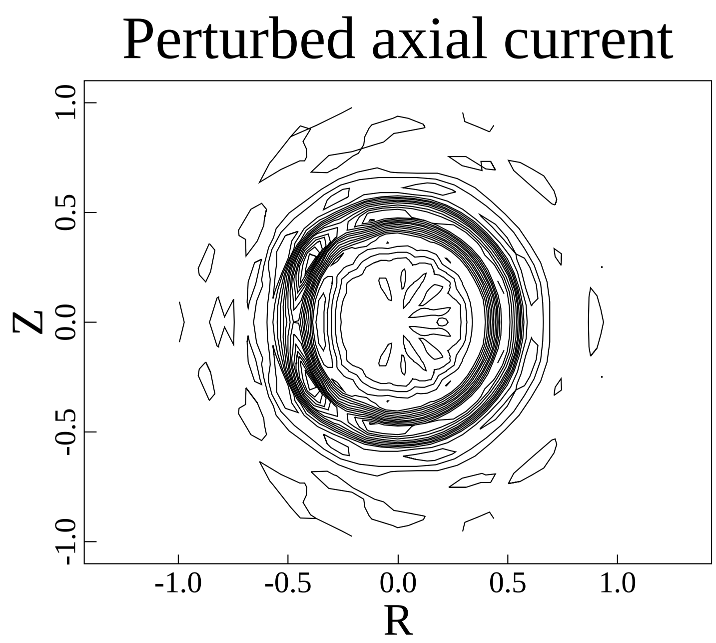
- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 1.80 \times 10^{-6}$

Current equilibration: Alfvén timescale

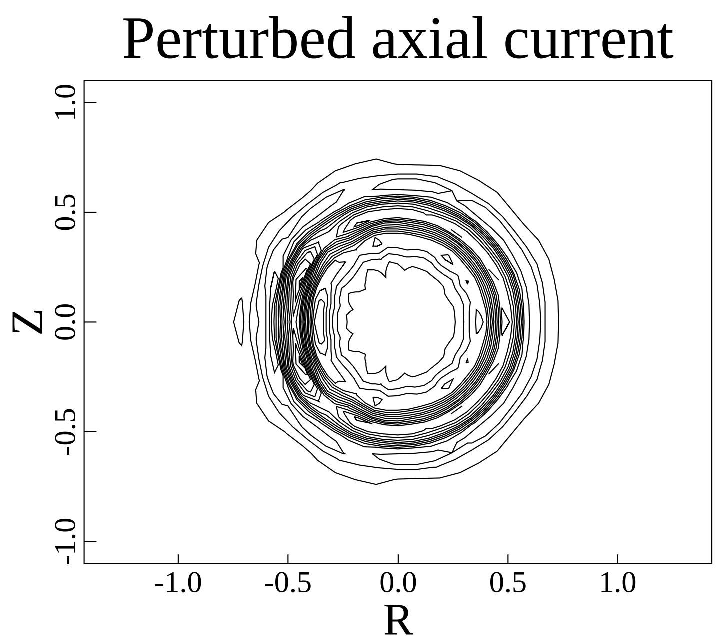
- Examine this process more closely by reducing the timestep and increasing λ_{rf}
- $\lambda_{\mathrm{rf}} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 1.90 \times 10^{-6}$

Current equilibration: Alfvén timescale

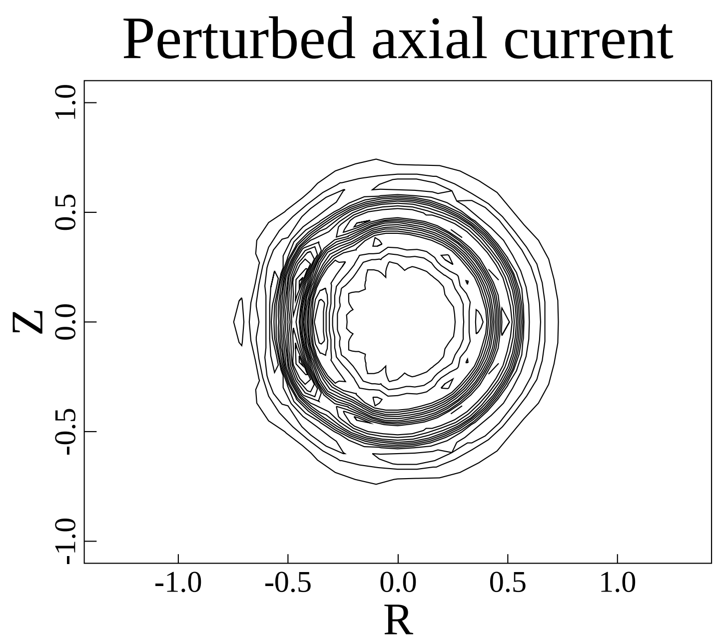
- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:



$t = 2.00 \times 10^{-6}$

Current equilibration: Alfvén timescale

- Examine this process more closely by reducing the timestep and increasing $\lambda_{\rm rf}$
- $\lambda_{\rm rf} = 16.0, \Delta t = 1.0 \times 10^{-7}, \tau_A = 8.3 \times 10^{-7}$:

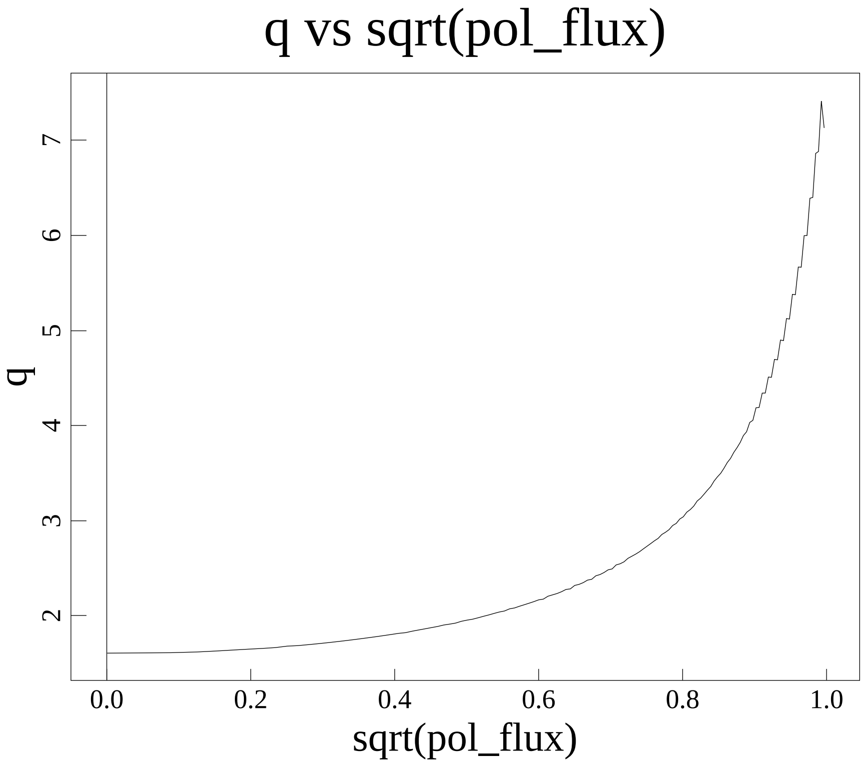


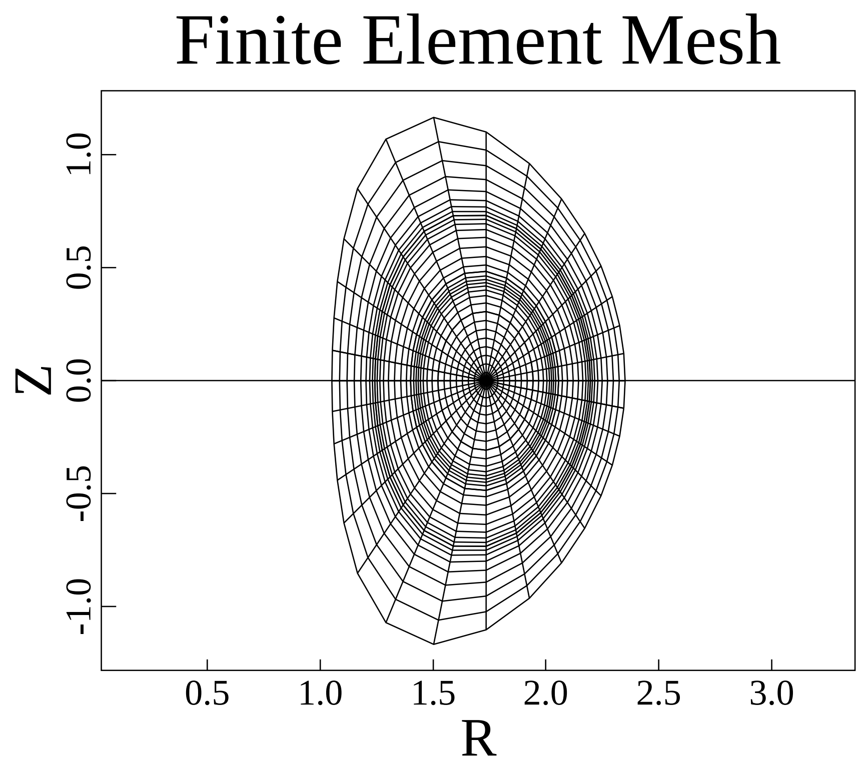
$t = 2.00{\times}10^{-6} = 2.4\tau_A$

- Current equilibrates over a flux surface on the Alfvén timescale.

Test this effect in realistic (DIII-D-like) geometry

- Grid packing used to resolve rational surfaces ($q = 2, q = 3$) more accurately

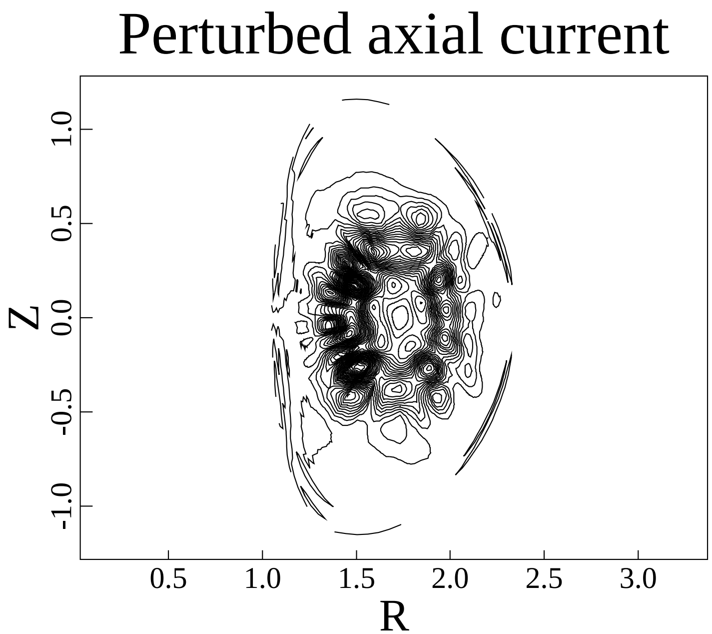




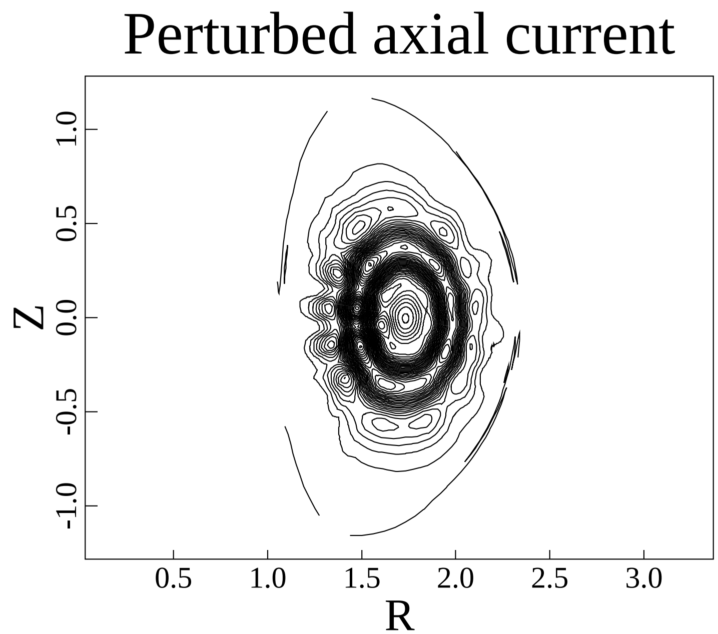
- Benchmark: DIII-D-like simulations of Sovinec *et al.* [J. Comp. Phys. **195**, 355 (2004)].

- Use axially symmetric $\mathbf{J}_{\rm rf}$ (a "ring" of current in the tokamak); examine effects on axially symmetric component of equilibrium ($n = 0$)

Current equilibration occurs in realistic geometries



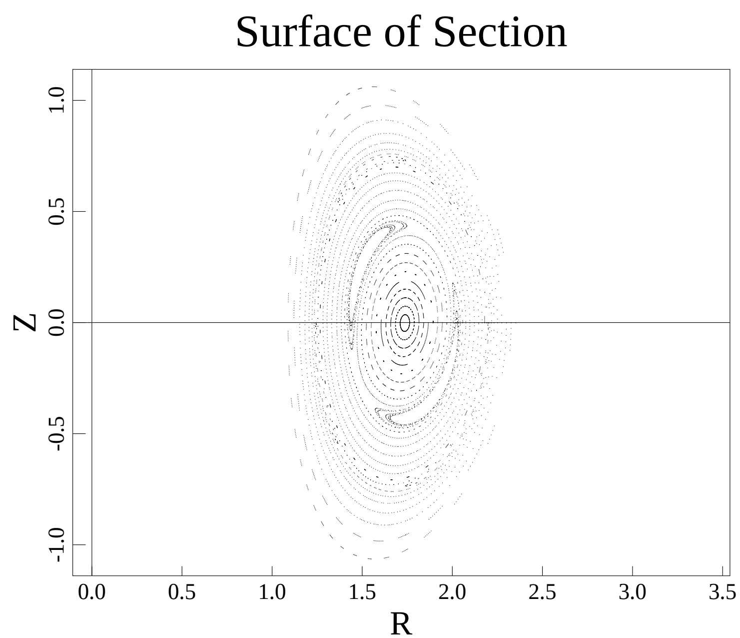
$t = 600\Delta t = 1000\tau_A$



$t = 700\Delta t = 1170\tau_A$

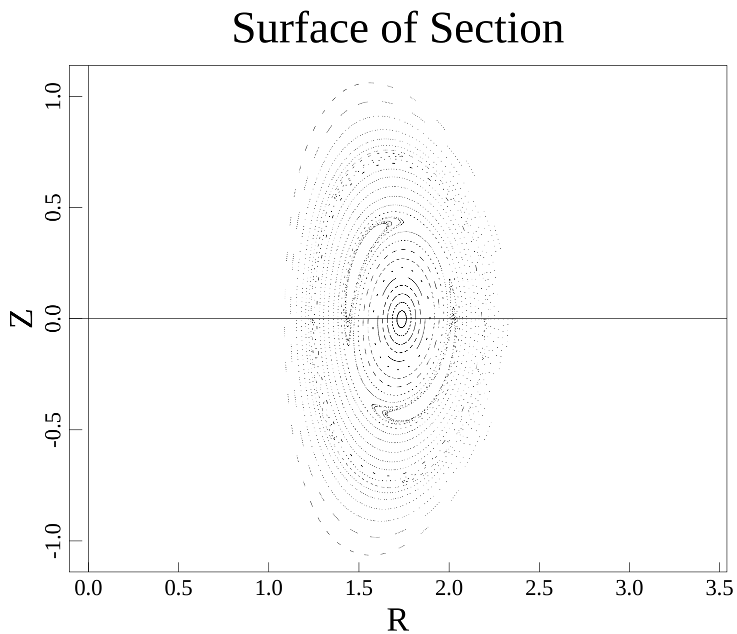
- Equilibration slower in realistic geometries; still fast compared to resistive diffusion time.
- $\lambda_{\mathrm{rf}} = 1.0, Z_{\mathrm{rf}} = 0, R_{\mathrm{rf}} = 2.0, w_{\mathrm{rf}} = 0.1$
- $t_{\mathrm{offset}} = 0.02, t_{\mathrm{build}} = 0.005, \Delta t = 1.0 \times 10^{-5}, \tau_R = 0.998, \tau_A = 6.0 \times 10^{-7}$
- $R/a \sim 3, S = 1.66 \times 10^6, Pr = 0.1$

(2,1) magnetic islands grow/saturate in the absence of RF term

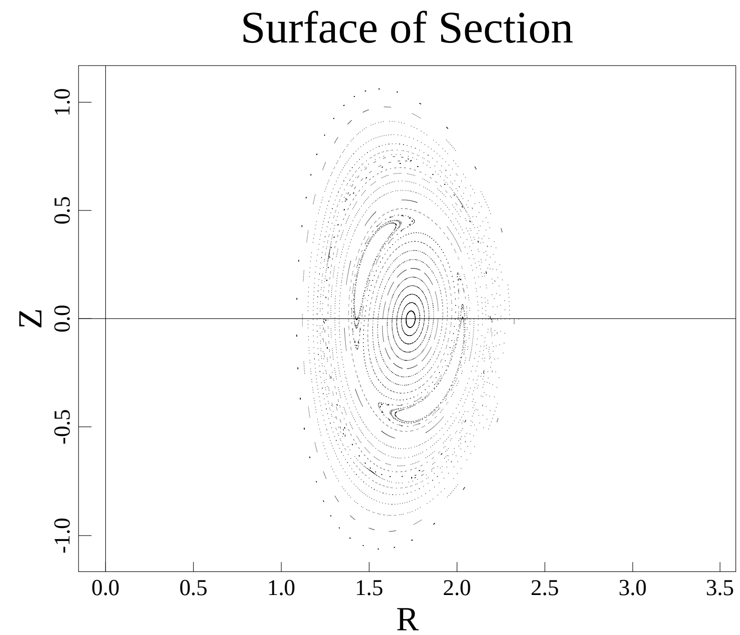


- Dominant nonlinear effect is $(2, 1)$ magnetic island
- Smaller island chain at $q = 3$ surface
- What happens when RF term is added?

RF effects influence island size . . . issues remain



No RF



RF

- Islands appear to be increasing slightly in size (wrong direction)
- Some issues with numerical stability of RF runs; working with Dalton Schnack and Carl Sovinec to address this
- Next steps: Investigation of effect of ad hoc current location/breadth on island size; non-axisymmetric sources, etc.