# Distribution functions for MHD, Nova-K in particular

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**SWIM Workshop**

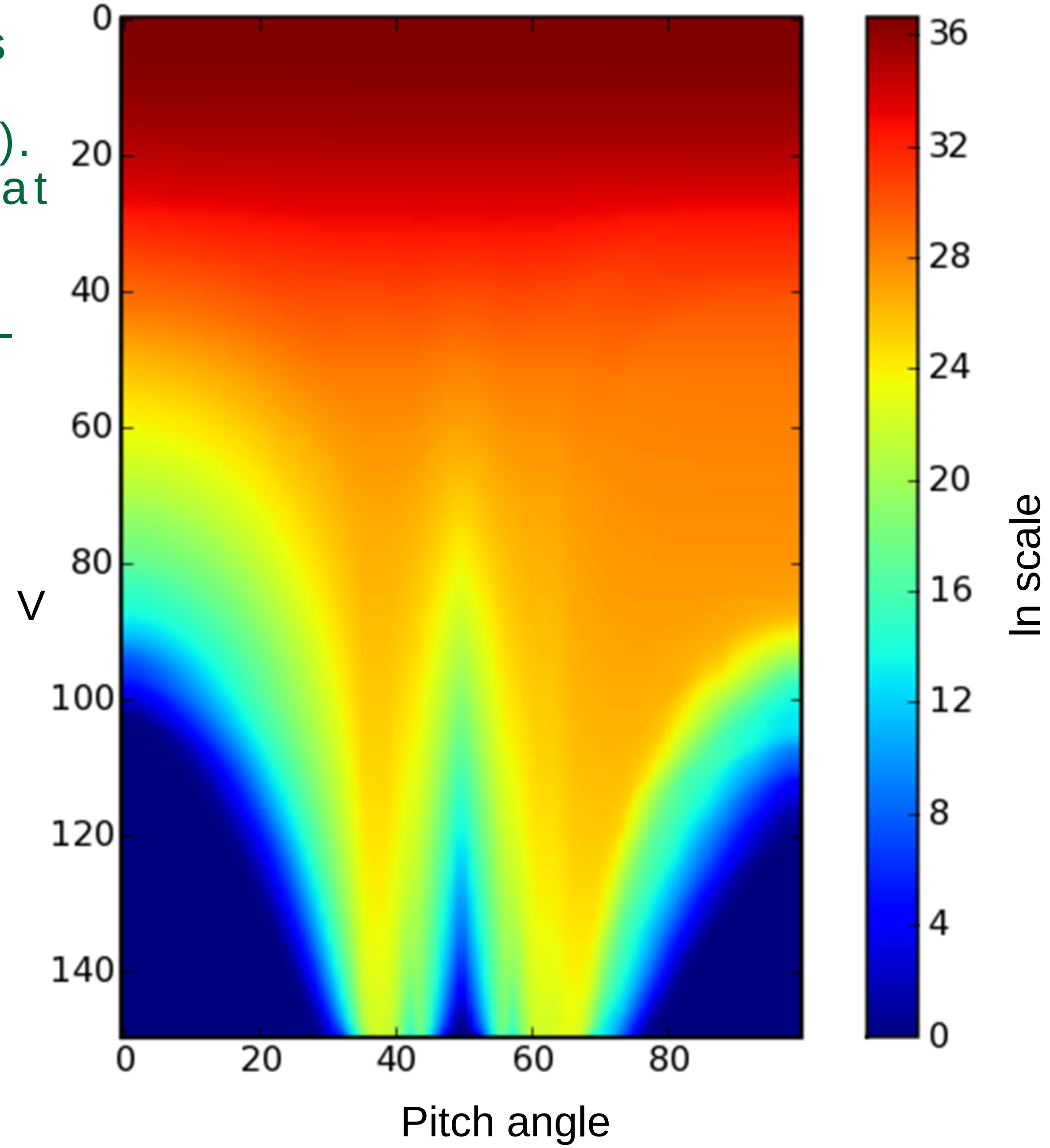
# TOPICS

- **Issues in coupling particle-based distributions to MHD and RF Simulations.**
- **SVD background.**
- **Outline of algorithm to implement coupling.**
- **Further discussion of issues is scheduled for Tuesday afternoon.**

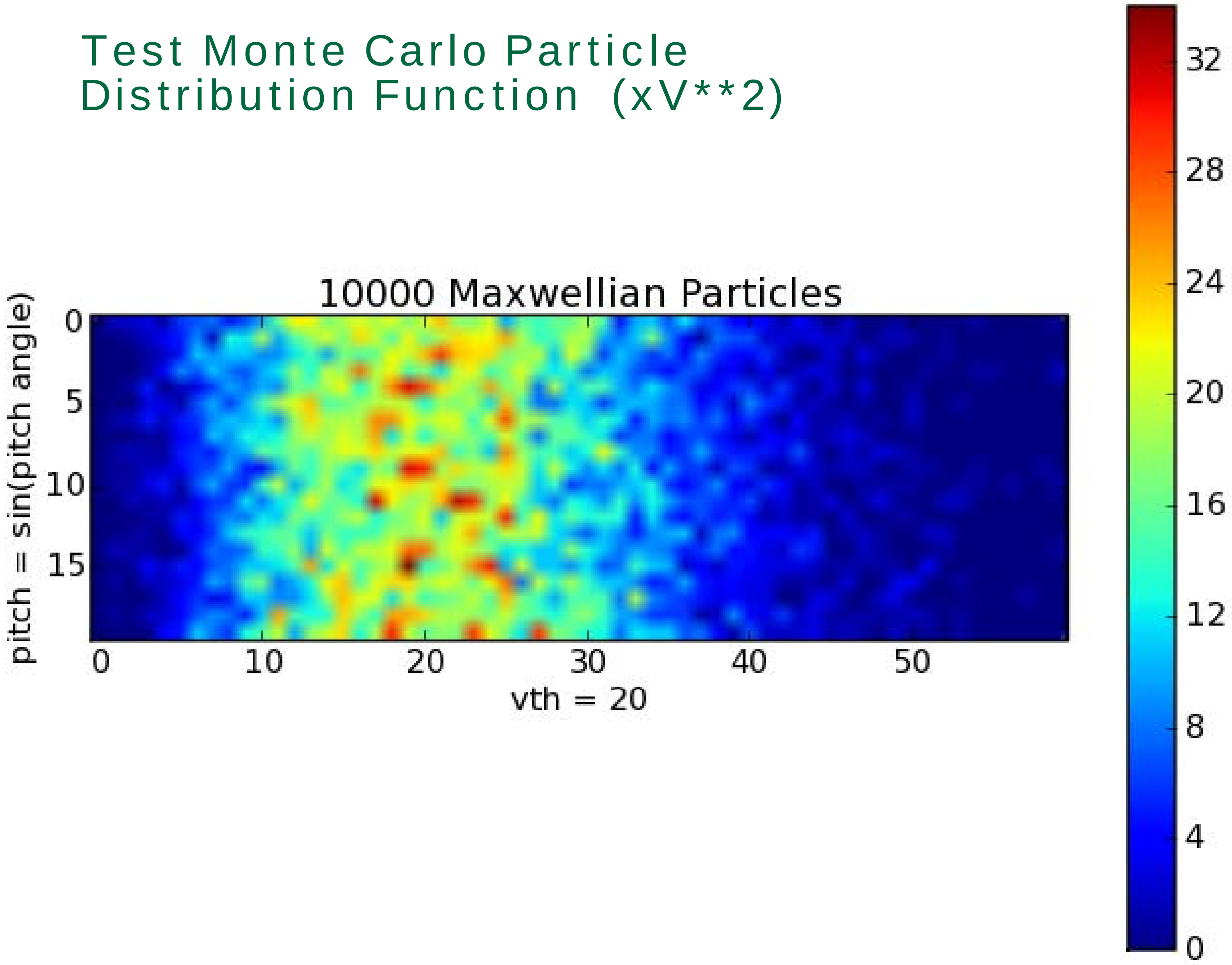
# Input Requirements of MHD and RF simulations

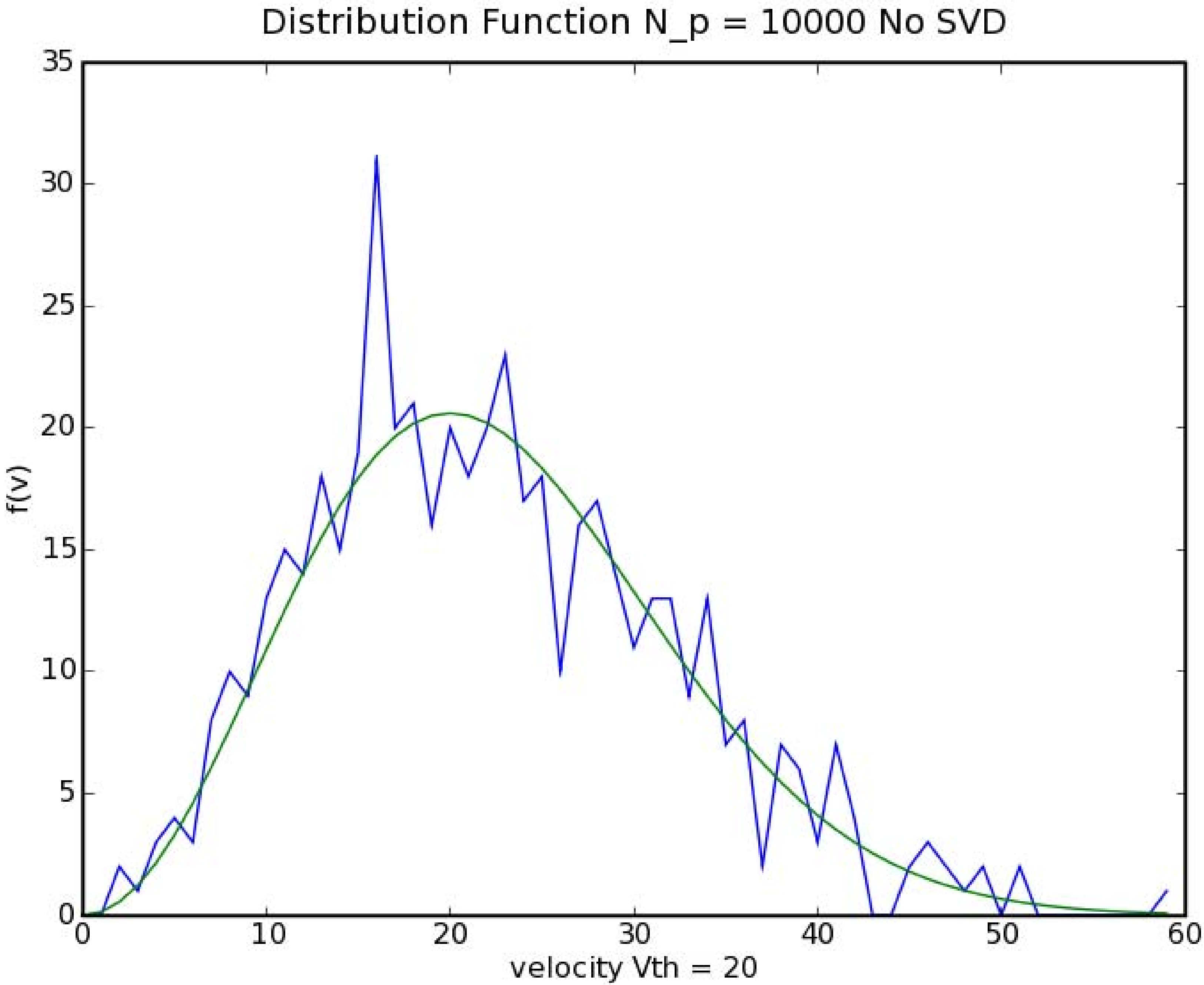
- **Moments of the distribution function are needed as inputs to MHD simulations, e.g.,** $\overleftrightarrow{P}_{i,j}\left(\vec{r}\right)=m\int v_i v_j f d^3v$ **and spatial gradients thereof.**
- **For MHD kinetic effects (Nova-K) and for RF, more localized properties are required. For example, Landau damping is proportional to terms like** $\int \frac{J^l\left(k_\perp \rho\right) J^l\left(k_\perp \rho\right)\vec{v}\cdot\vec{\nabla}_v f}{\omega - k_{||}v_{||} - l\omega_c} d^3v$**. (The response is evaluated as the Cauchy principal-parts integral plus a delta function.)**
- **The Bessel functions and cyclotron-harmonic resonances are likely not required for MHD.**
- **Monte Carlo simulations for** $f$ **with thousands to tens of thousands of particles per radial grid point are likely to provide sufficient statistics for the MHD. However, sensitivity to anisotropy and need for Landau damping may require processing statistical distributions to avoid long simulation times for large particle number simulations.**

C-Mod distribution function (CQL3D) at half-radius. The X-axis is the pitch, and the Y-Axis is velocity (speed). Note the “rabbit” ears at the trapped-passing boundary. (From a self-consistent AORSA-CQL3D simulation of minority heating.)



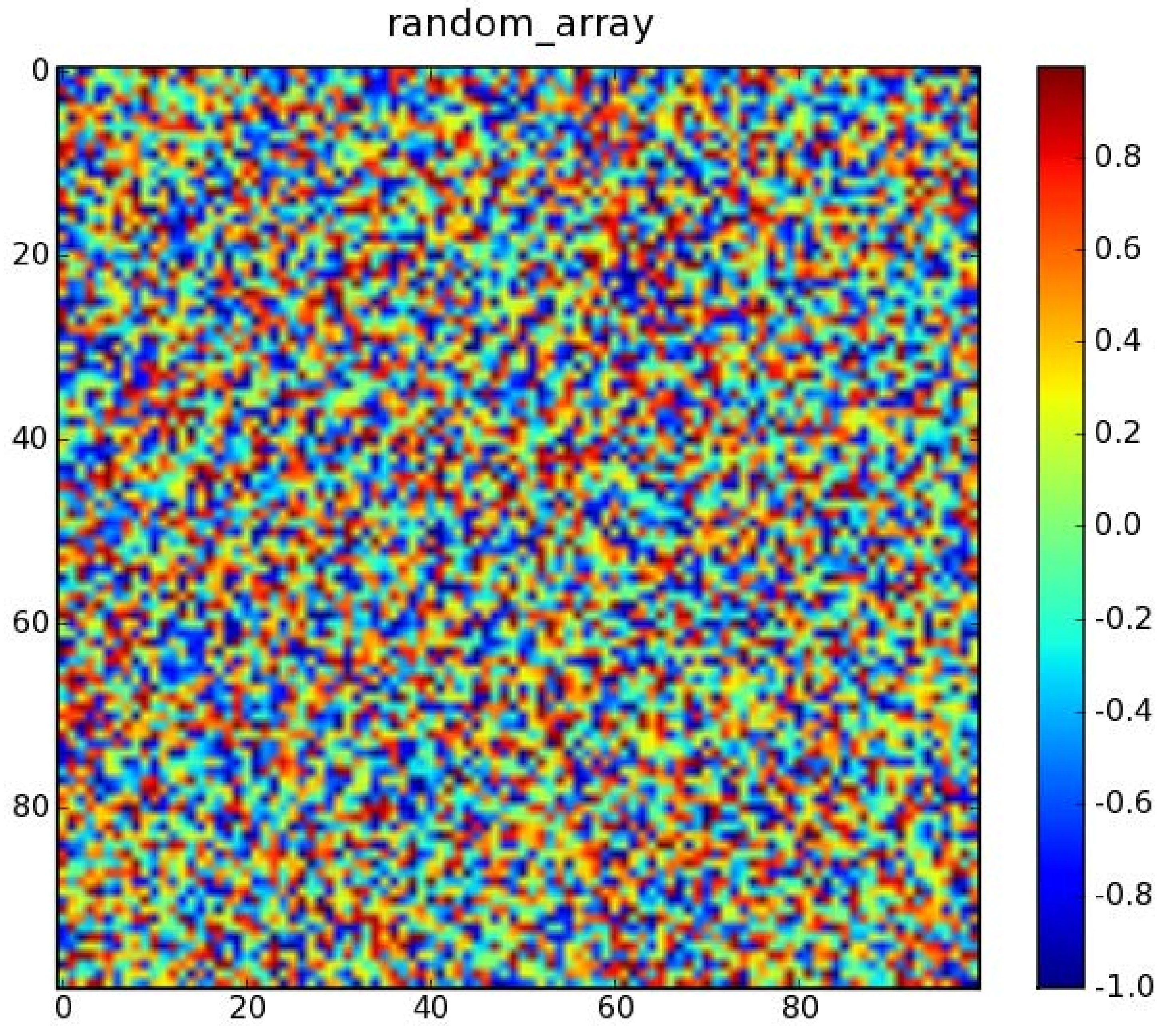
# Test Monte Carlo Particle Distribution Function (xV**2)

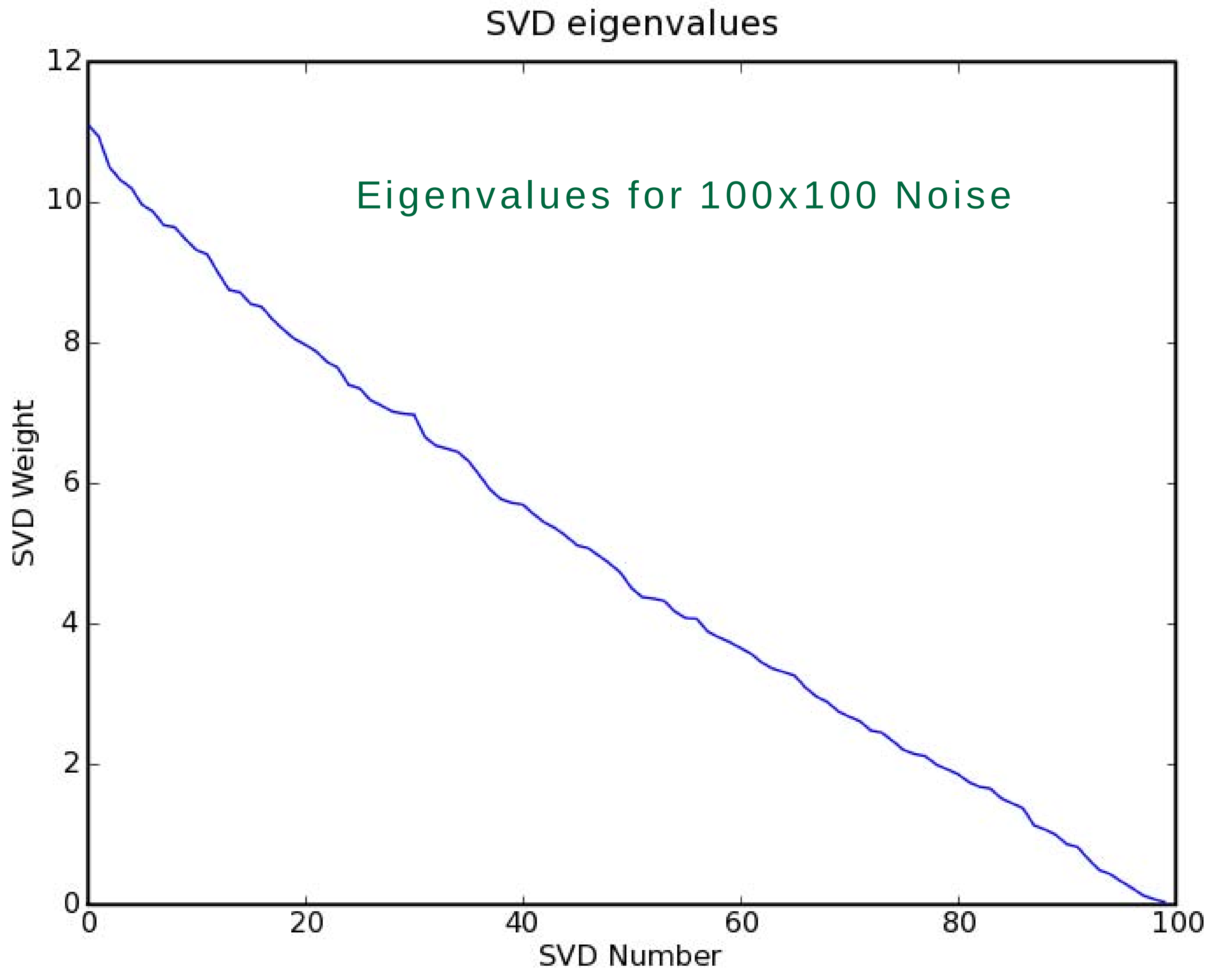



Distribution Function N_p = 10000 No SVD
f(v)
velocity Vth = 20
0
5
10
15
20
25
30
35
0
10
20
30
40
50
60

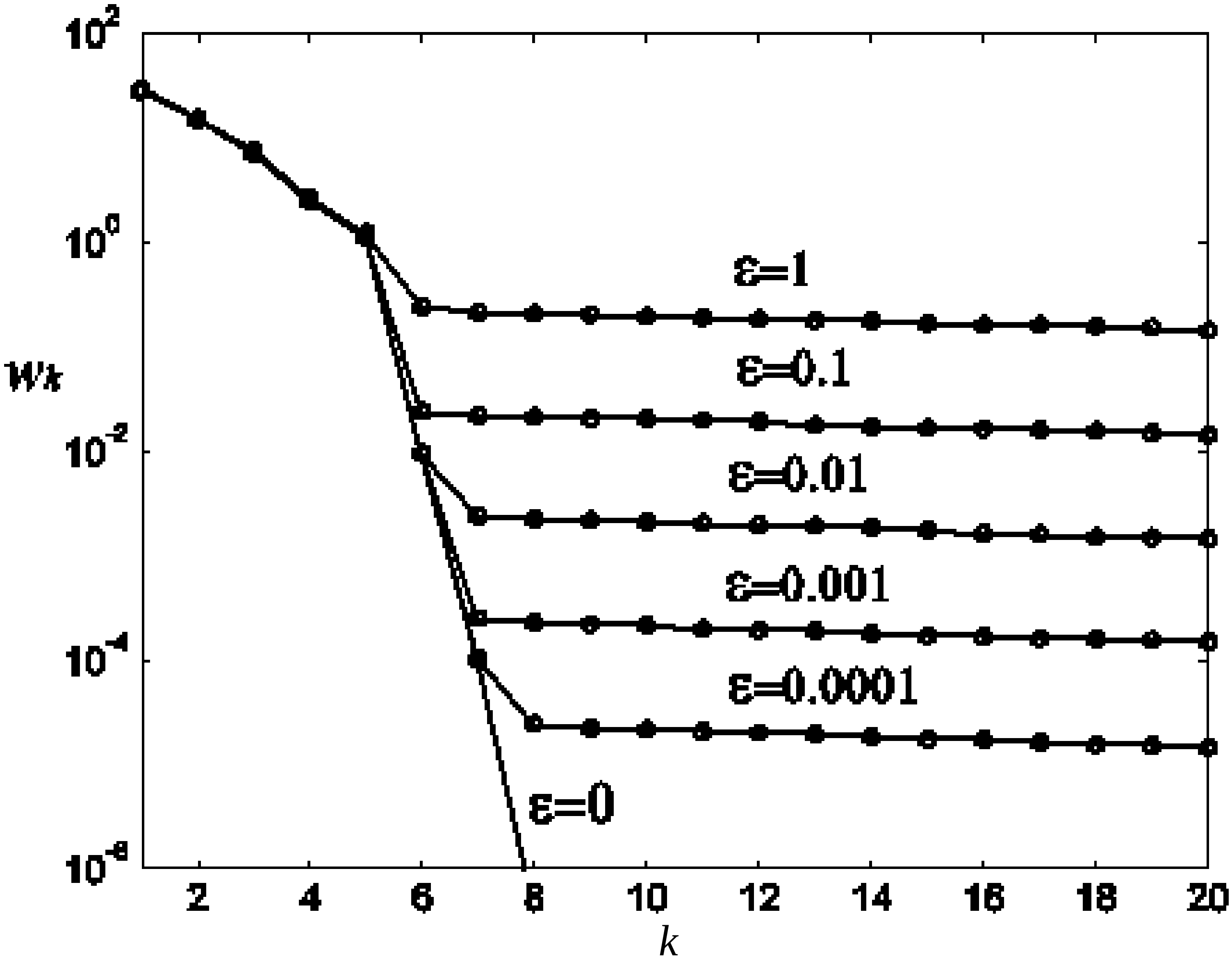
# Singular Value Decomposition (SVD): Background

- **For an $m \times n$ matrix $\overleftrightarrow{A}$, the SVD is given by $\overleftrightarrow{A} = \overleftrightarrow{U} \cdot \overleftrightarrow{\Sigma} \cdot \overleftrightarrow{V}^H$, where $\overleftrightarrow{U}$ and $\overleftrightarrow{V}$ are orthogonal matrices ($m \times m$ and $n \times n$) respectively, and $\overleftrightarrow{\Sigma}$ an $m \times n$ "diagonal" matrix with values $\lambda_{ii}$. By convention, the $\lambda_{ii}$ are ordered by decreasing magnitudes.**
- **A rank $r$ approximation to $\overleftrightarrow{A}$, $\overleftrightarrow{A}_r$, may be generated from $\overleftrightarrow{U} \cdot \overleftrightarrow{\Sigma}_r \cdot \overleftrightarrow{V}^H$ where $\overleftrightarrow{\Sigma}_r$ $r$ retains the first $r$ of the $\lambda_{ii}$.**
- **Two properties are of interest:**
  - **$\overleftrightarrow{U}$ and $\overleftrightarrow{V}^H$ are an optimal tensor-product basis set for $\overleftrightarrow{A}$.**
  - **"Noise" in $\overleftrightarrow{A}$ manifests as a plateau in the SVD spectrum.**

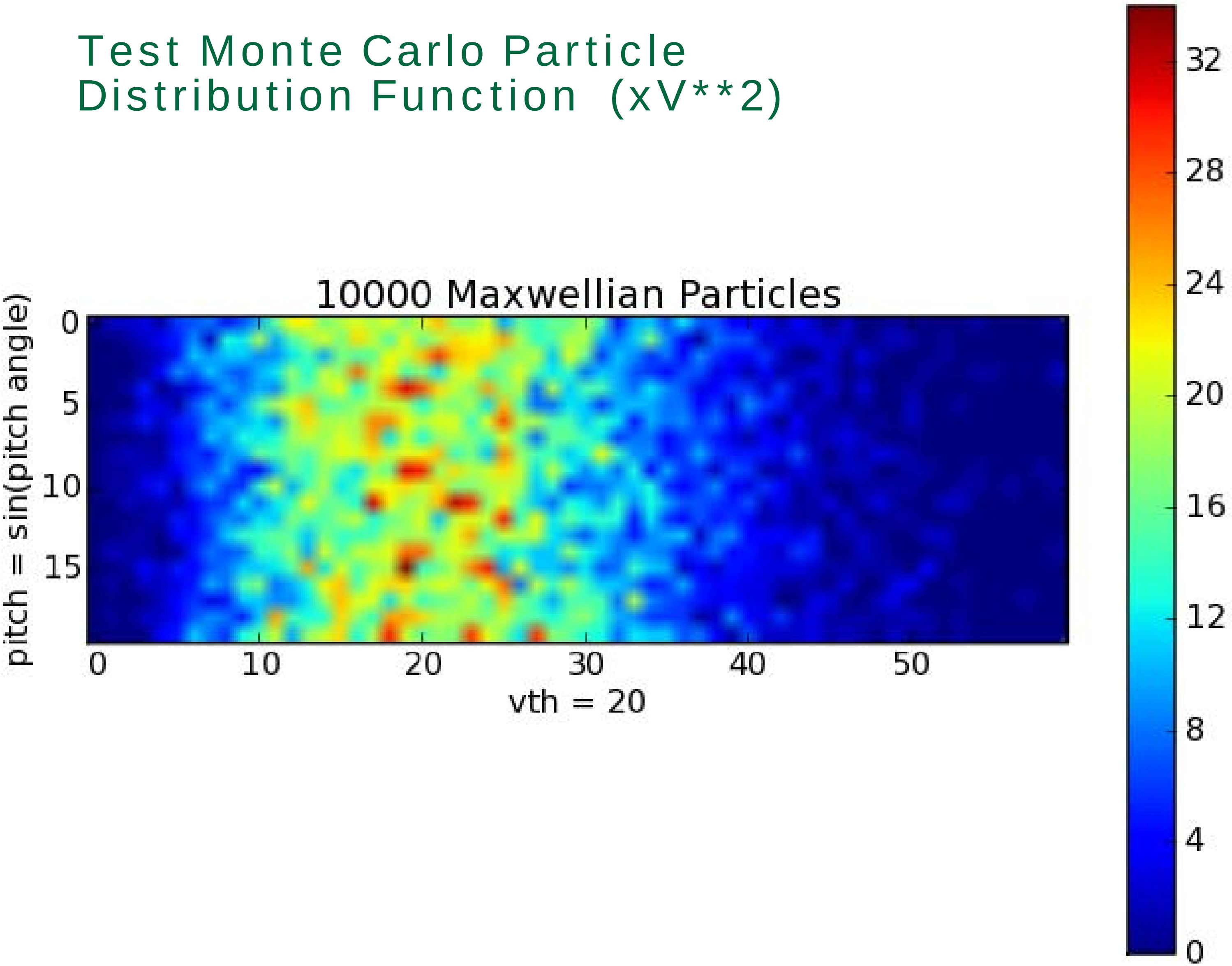

random_array
0
20
40
60
80
0
20
40
60
80
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0

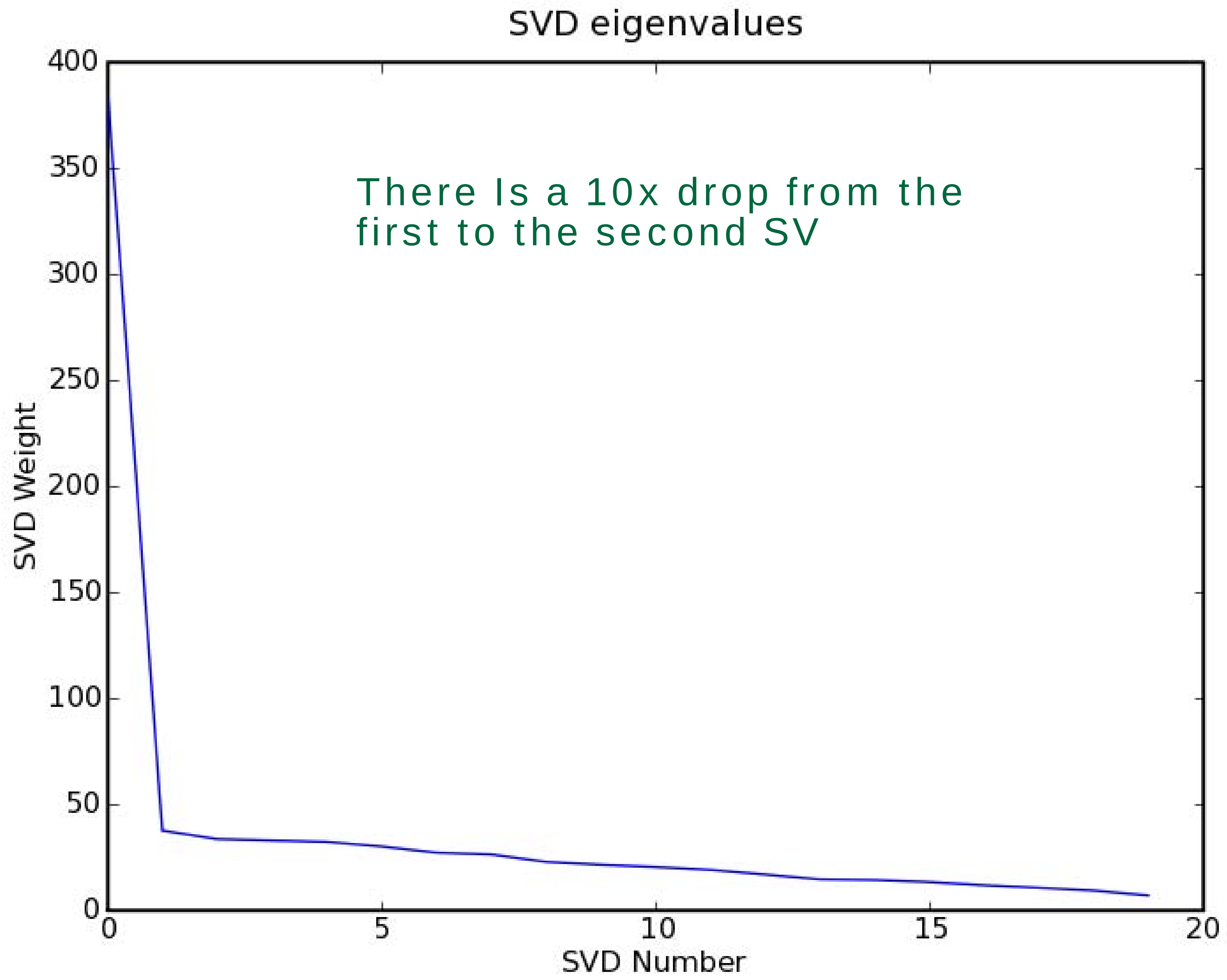

SVD eigenvalues
Eigenvalues for 100x100 Noise
SVD Weight
SVD Number
0
2
4
6
8
10
12
0
20
40
60
80
100

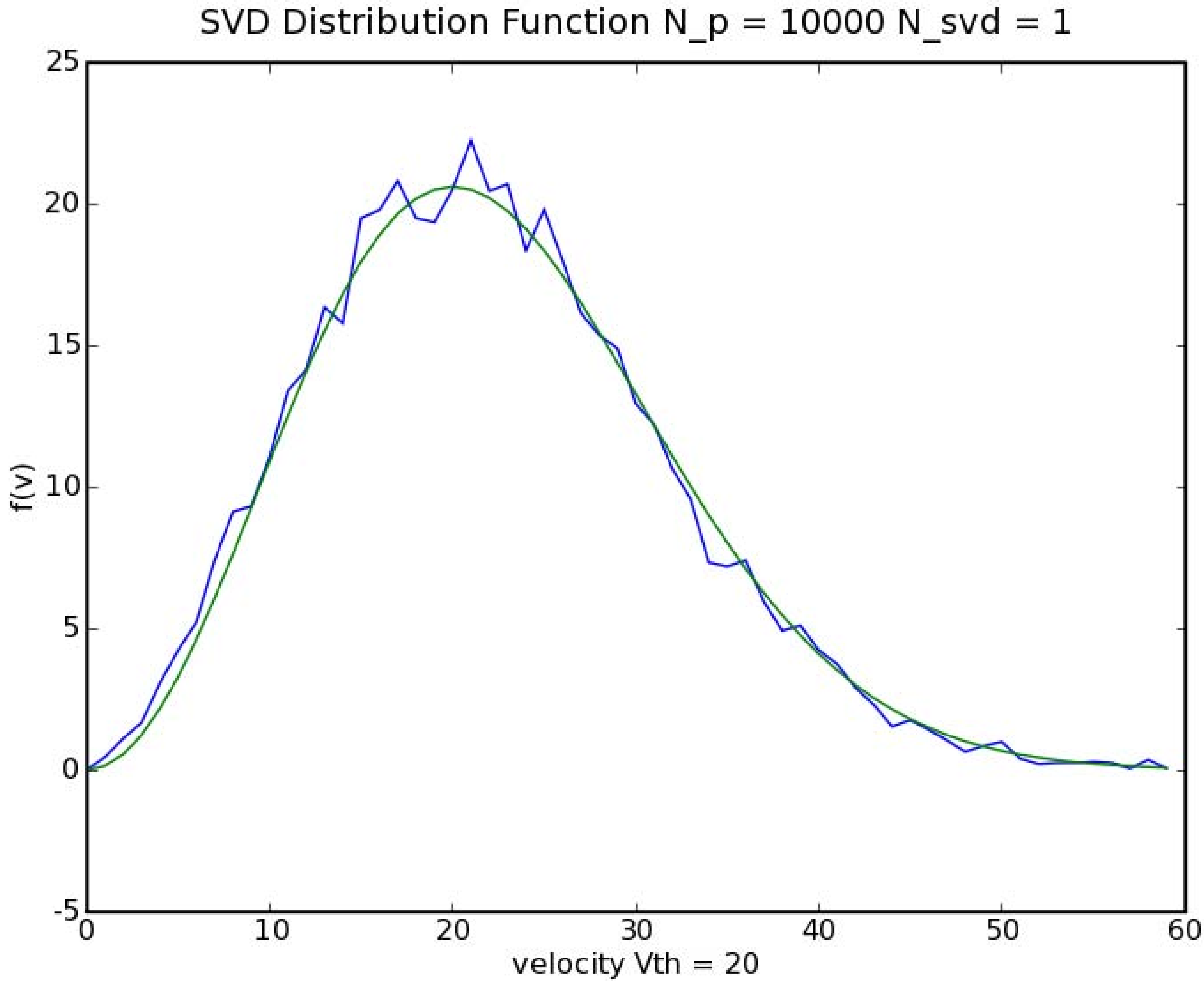
# When Noise Is Introduced, the Energy (SVD weight**2) Exhibits a Plateau (k = # of SVD Not Retained)

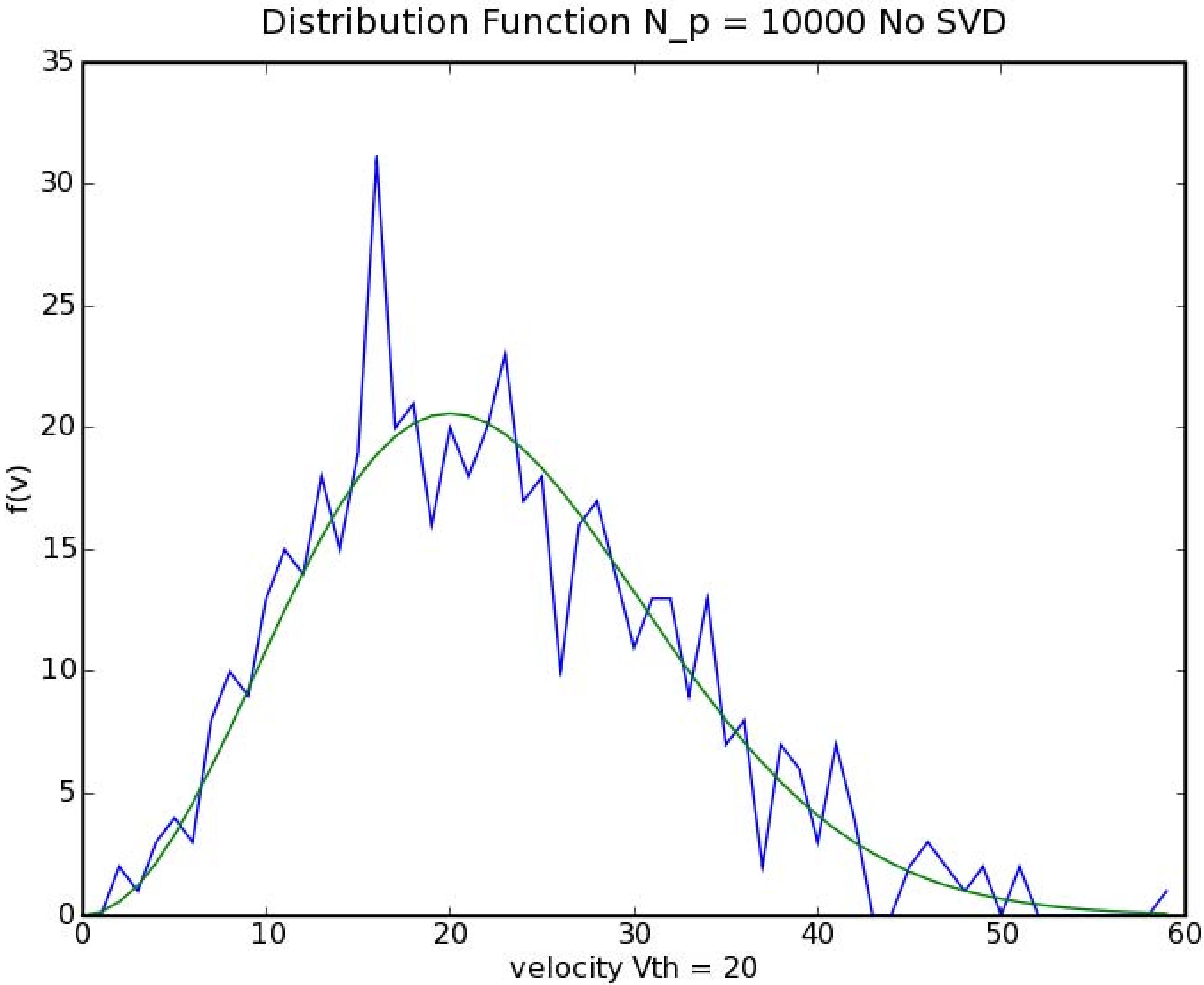


# Test Monte Carlo Particle Distribution Function (xV**2)

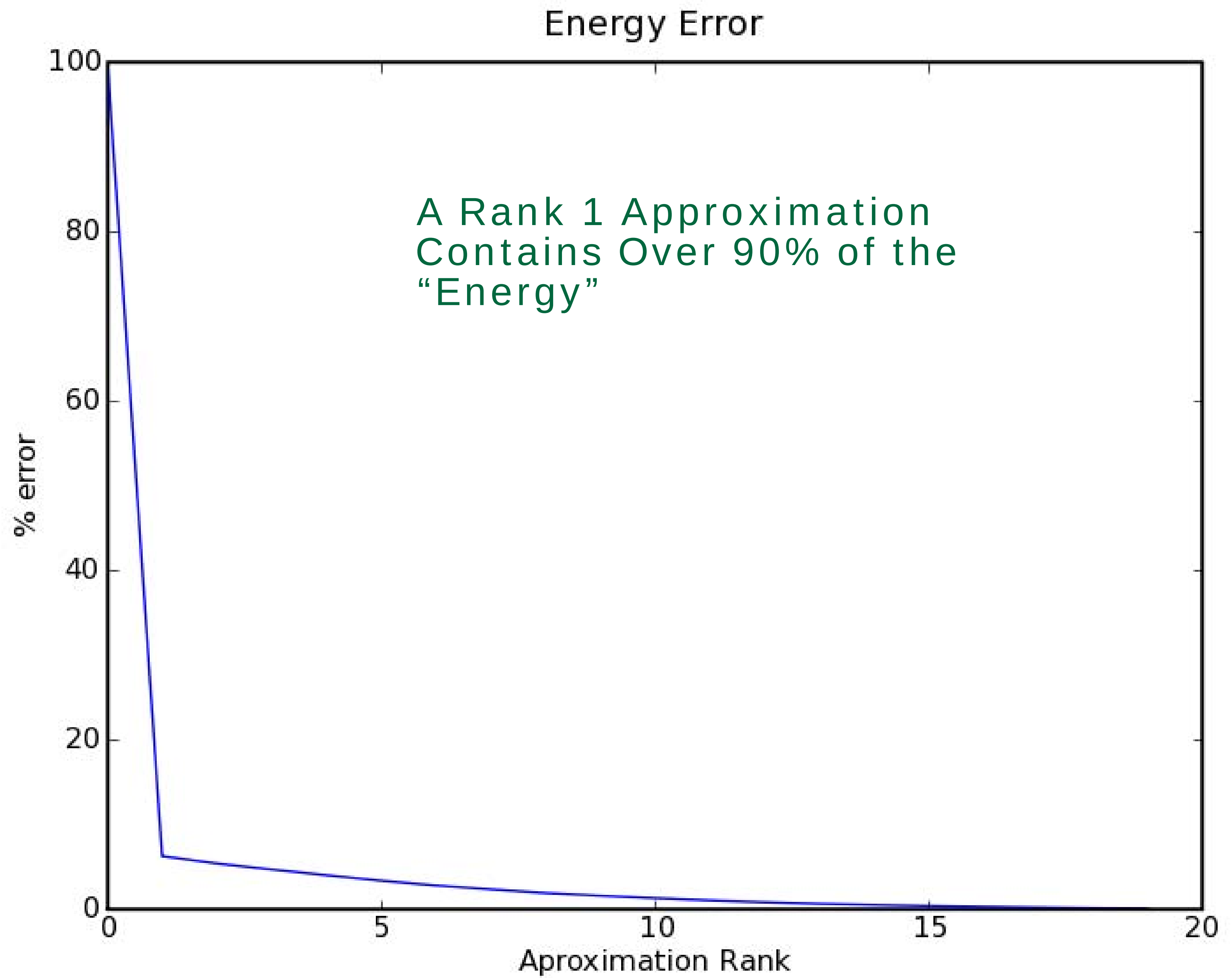



SVD eigenvalues
There Is a 10x drop from the
first to the second SV
SVD Weight
SVD Number
400
350
300
250
200
150
100
50
0
0
5
10
15
20

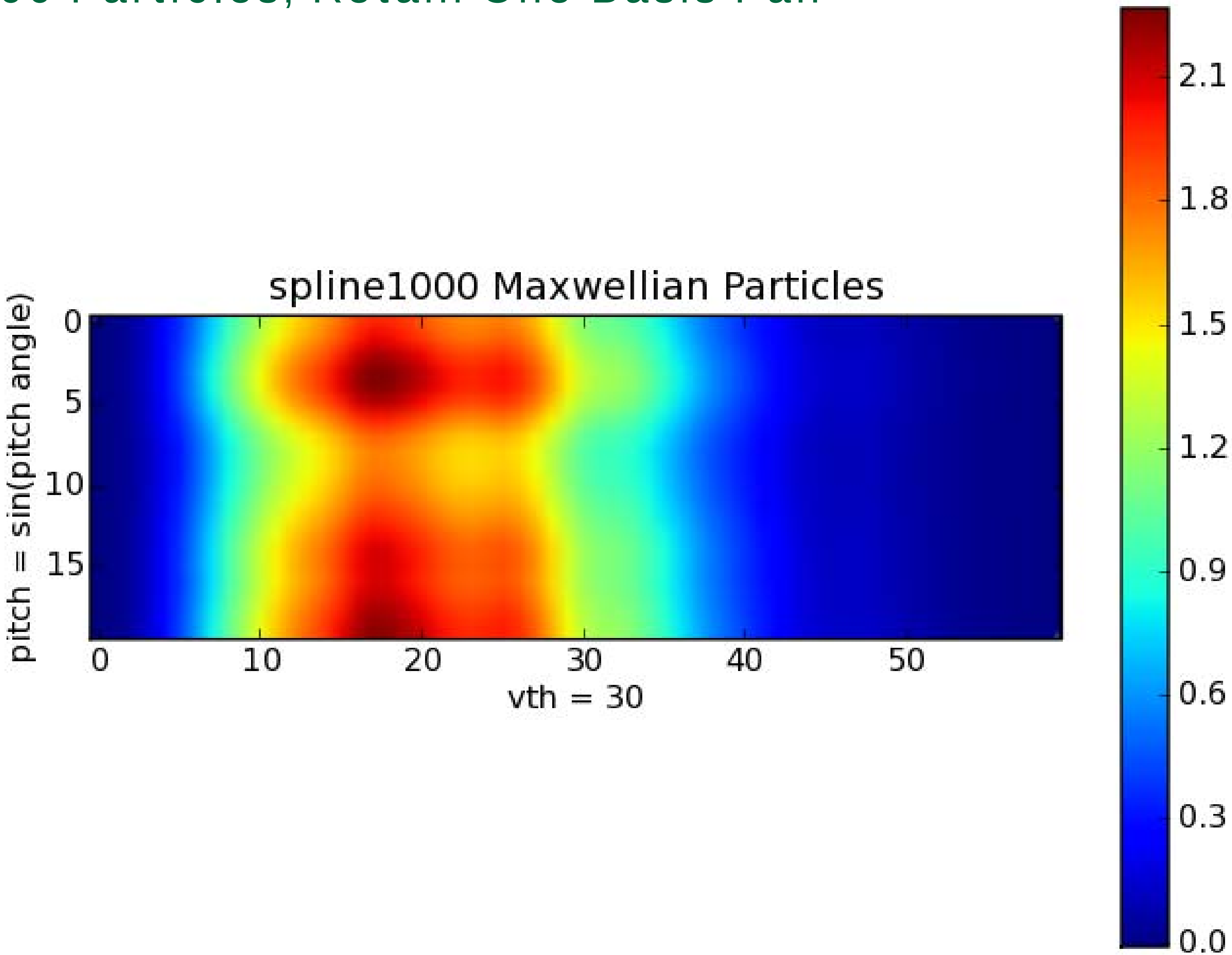

SVD Distribution Function N_p = 10000 N_svd = 1
f(v)
velocity Vth = 20
25
20
15
10
5
0
-5
0
10
20
30
40
50
60

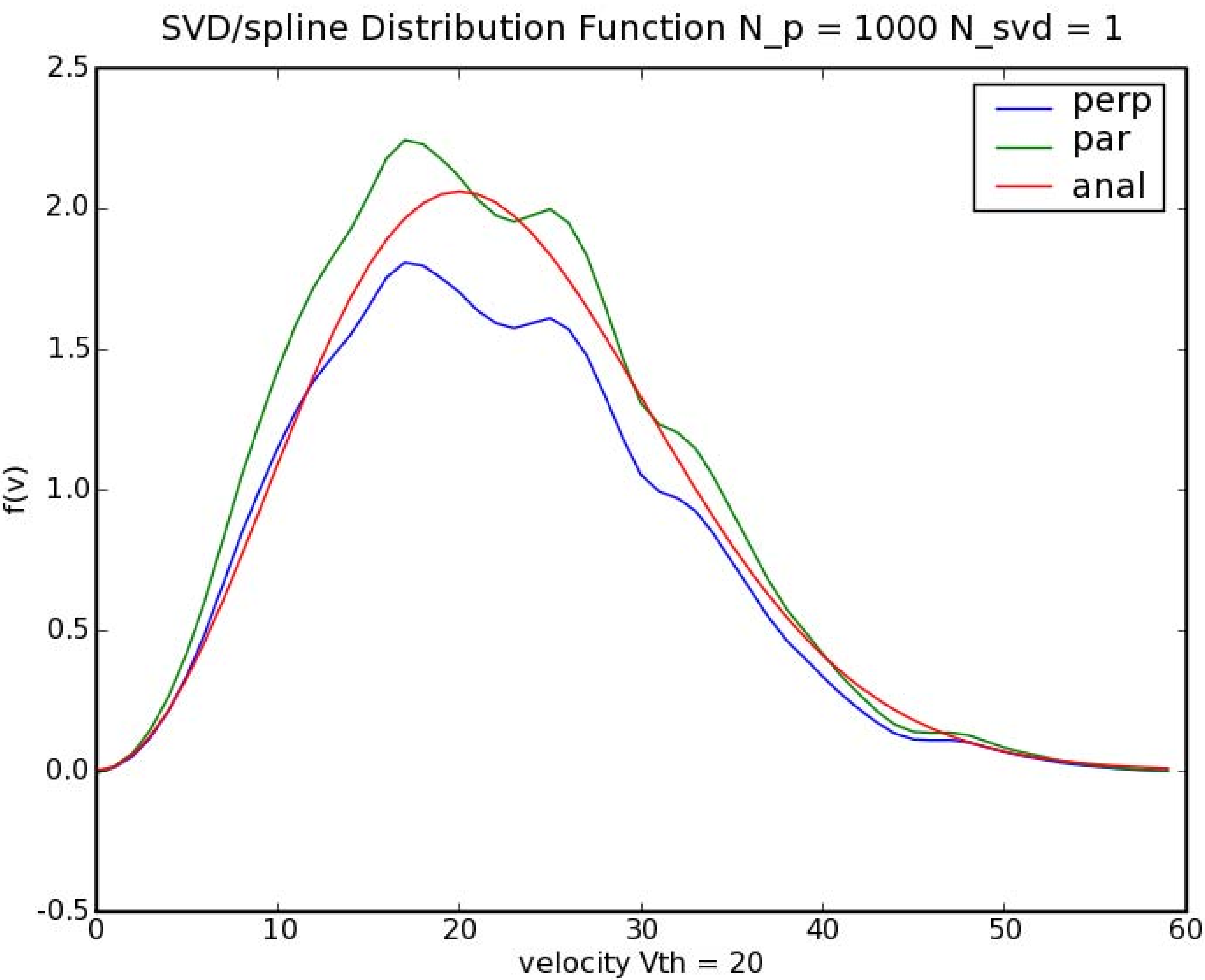


Distribution Function N_p = 10000 No SVD
f(v)
velocity Vth = 20
0
5
10
15
20
25
30
35
0
10
20
30
40
50
60

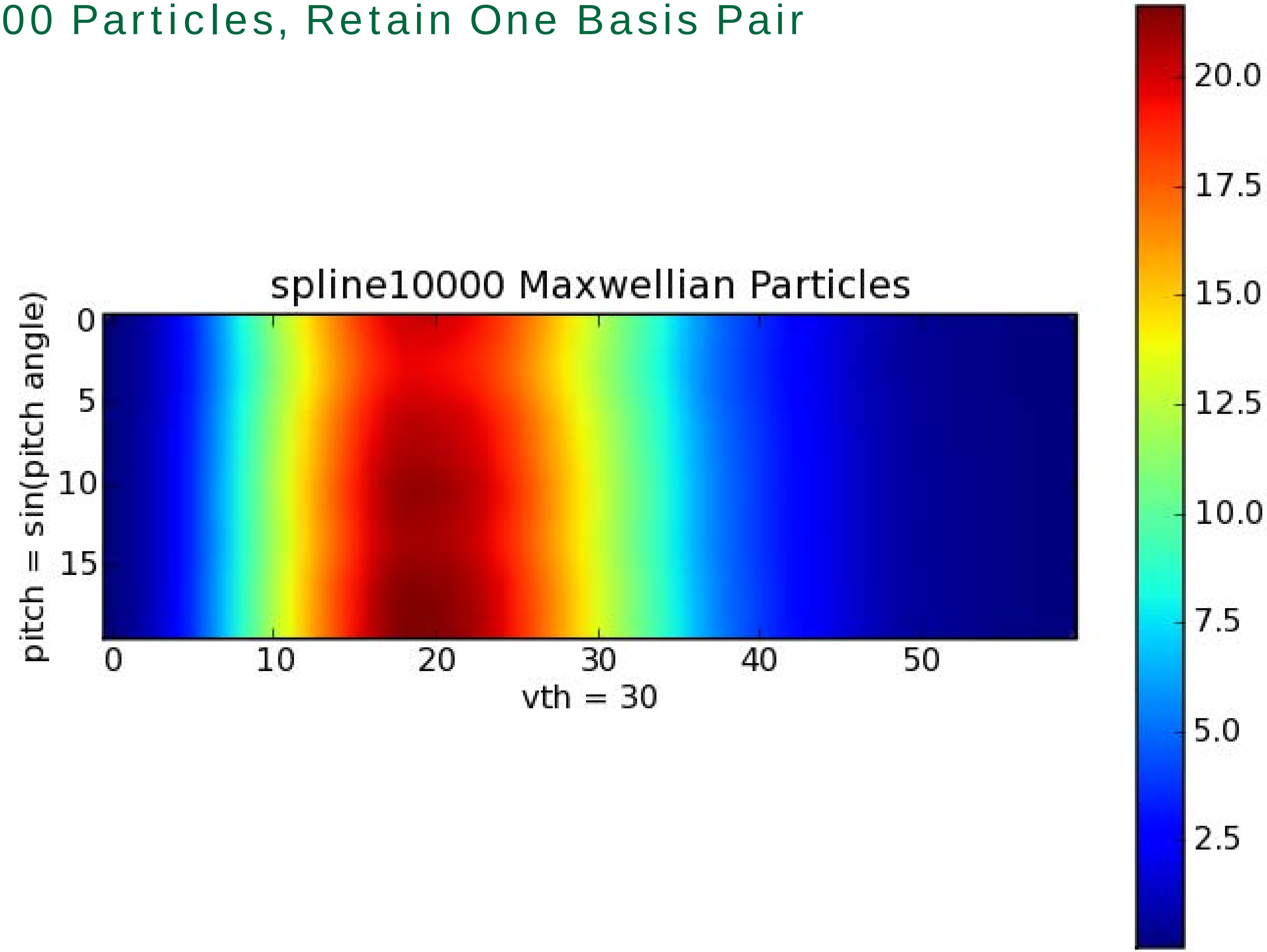

Energy Error
A Rank 1 Approximation
Contains Over 90% of the
"Energy"
% error
Aproximation Rank
100
80
60
40
20
0
0
5
10
15
20

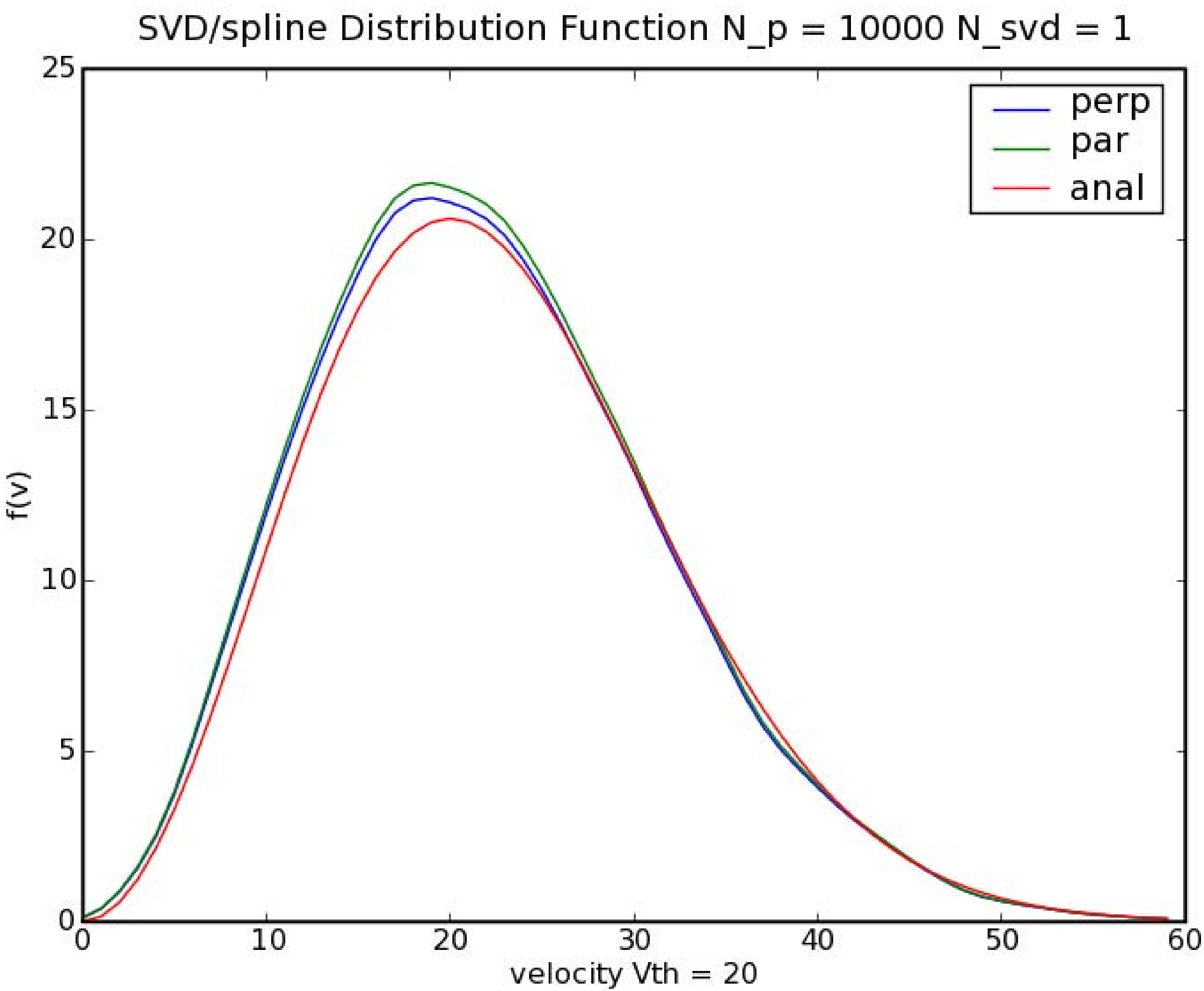
# Maxwellian Reconstruction Using Spline SVD Basis Vectors: 1000 Particles, Retain One Basis Pair




SVD/spline Distribution Function N_p = 1000 N_svd = 1
perp
par
anal
f(v)
velocity Vth = 20
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
0
10
20
30
40
50
60

# Maxwellian Reconstruction Using Spline SVD Basis Vectors: 1000 Particles, Retain One Basis Pair





SVD/spline Distribution Function N_p = 10000 N_svd = 1
perp
par
anal
f(v)
velocity Vth = 20
0
5
10
15
20
25
0
10
20
30
40
50
60

# Algorithm Outline

- **Generate MC distribution from 4D quasi linear diffusion operator.**
- **Bin 4D particles, $\left(r,\theta,v,\lambda\right)$, in constants of motion, matrix $\left(P_{\varphi},E,\mu\right)$, then transform to coordinates that are something like energy and pitch.**
- **Process and then transform to 4D space for RF and/or MHD.**
- **Advance RF and/or MHD.**
- **Calculate 4D QL operator from fields.  Options are possible for dynamical evaluation of QL operator, or for direct particle evolution from fields.**

## SVD Issues

- **Dependent variables:  maximally-separable variables should be divined.  The better the variable separation the better the low-rank approximation and separation from noise.**
- **Smoothing  may not be necessary for radial coordinate.**
- **Do we need to weight the tail so as not to loose tail anisotropy because of higher energy in the “bulk?**
- **Can the rank of optimum approximation be automated?**
- **Are there better approaches?**