UT-BATTELLE

Neoclassical Effects and NCLASS in IPS

W.A. Houlberg
ORNL

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Neoclassical Effects in NCLASS

Particle balances: $$\frac{\partial n_j}{\partial t} = -\nabla \cdot \left(\Gamma_j^{NC} + \Gamma_j^{T} \right) + S_{pj}$$

- **Neoclassical particle flux provided in several optional forms for all species – total flux, diffusive and convective (D and V), full transport matrix**

Energy balances: $$\frac{\partial \left(\frac{3}{2} n_j T_j \right)}{\partial t} = -\nabla \cdot \left[\left(q_j^{NC} + q_j^{T} \right) + \frac{3}{2} \Gamma_j \right] + S_{Ej} + \ldots$$

- **Neoclassical heat flux provided in several optional forms for all species – total flux, diffusive and convective (χ and V_χ), full transport matrix** $J_{F\|} = G_F n_F v_{F\|}$

Ohm's law: $$\langle E \cdot B \rangle = \eta_\| \left(\langle J \cdot B \rangle - \langle J_{BS} \cdot B \rangle - \langle J_F \cdot B \rangle - \langle J_{RF} \cdot B \rangle \right)$$

- **Parallel electrical resistivity**
- **Bootstrap current**
- **Plasma current shielding factor for fast ions** $J_{F\|} = G_F Z_F n_F v_{F\|}$

Plasma rotation velocities within each flux surface: $$\vec{v}_j = K_j(\rho)\vec{B} + R\Omega_j(\rho)\hat{e}_\phi$$

- **Poloidal rotation velocity of all species**
- **Relative parallel/toroidal rotation velocities (bulk rotation undetermined)**

NCLASS Solves for a set of Velocity Moments of the Flows Within a Flux Surface

Neoclassical problem is generalized by addition of external forces

NCLASS solves a set of parallel force balance equations for each species – fluid equations generated by the odd velocity moments of the plasma kinetic equation:

$$\langle \vec{B} \cdot \vec{\nabla} \cdot \vec{\vec{P}}_{\alpha,j} \rangle = \langle \vec{F}^{\,c}_{\alpha,j} \cdot \vec{B} \rangle + \langle \vec{F}^{\,s}_{\alpha,j} \cdot \vec{B} \rangle \qquad \alpha = 1,2,\ldots,n;\ j = species$$

Viscosity | Friction | External forces

In combination with the radial force balances:

$$\langle B^2 \rangle \hat{u}_{\theta,1,j} = \hat{u}_{\|,1,j} + \frac{\mu_0 F}{\Psi'} \frac{p_j'}{e_j n_j} + \frac{\mu_0 F}{\Psi'} \Phi_E'$$

$$\langle B^2 \rangle \hat{u}_{\theta,2,j} = \hat{u}_{\|,2,j} + \frac{\mu_0 F}{\Psi'} \frac{k T_j'}{e_j}$$

$$\langle B^2 \rangle \hat{u}_{\theta,\alpha,j} = \hat{u}_{\|,\alpha,j} \qquad \alpha > 2$$

To obtain the α=1-n moments of the velocities within a flux surface

Recent NCLASS Developments

Standard neoclassical solutions only need two moments n=2, but other external forces need higher moments

- For example, the resonant Coulomb interaction between fast ions and thermal electrons requires n~10-15 to accurately calculate the electron shielding current for neutral beam current drive

Extended the calculation of the Coulomb friction matrix to arbitrary order, α=n (practical up to n~15 in double precision)

- Uses recursion relation to generate polynomial moments, which are then converted to Laguerre moments
- Validated against analytic models for α=1-3

Extended the calculation of the viscosity matrix to arbitrary order

- Recursion relation to generate polynomial moments for viscosity in banana regime was used to check direct numerical integration
- Direct numerical integration uses for general viscosities

Progress to be reported at APS meeting

Memo written on general treatment of flows, 'Rotation in multiple species axisymmetric plasmas'

Algorithms and Methods are Highly Developed

Number of variables:

- **2n*2k, where n is the number of moments and k the number of species**

Friction matrix:

- **Four dimensional (nxn)x(kxk) where n is the number of moments and k the number of species**

Viscosity matrix:

- **Three dimensional (nxn)xk**

Solution technique

- **k includes all charge states, so cases that include all chare states of higher Z impurities lead to large arrays**
- **Broken down into (nxn)x(mxm) submatrices where m represents each ion using reduced charge state method of generating eigenvectors for each charge state**
  - **e.g, six charge states of carbon collapses 6x6 submatrix to a single element**