

On neoclassical and rf closures - Application to neoclassical tearing modes

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Thesis

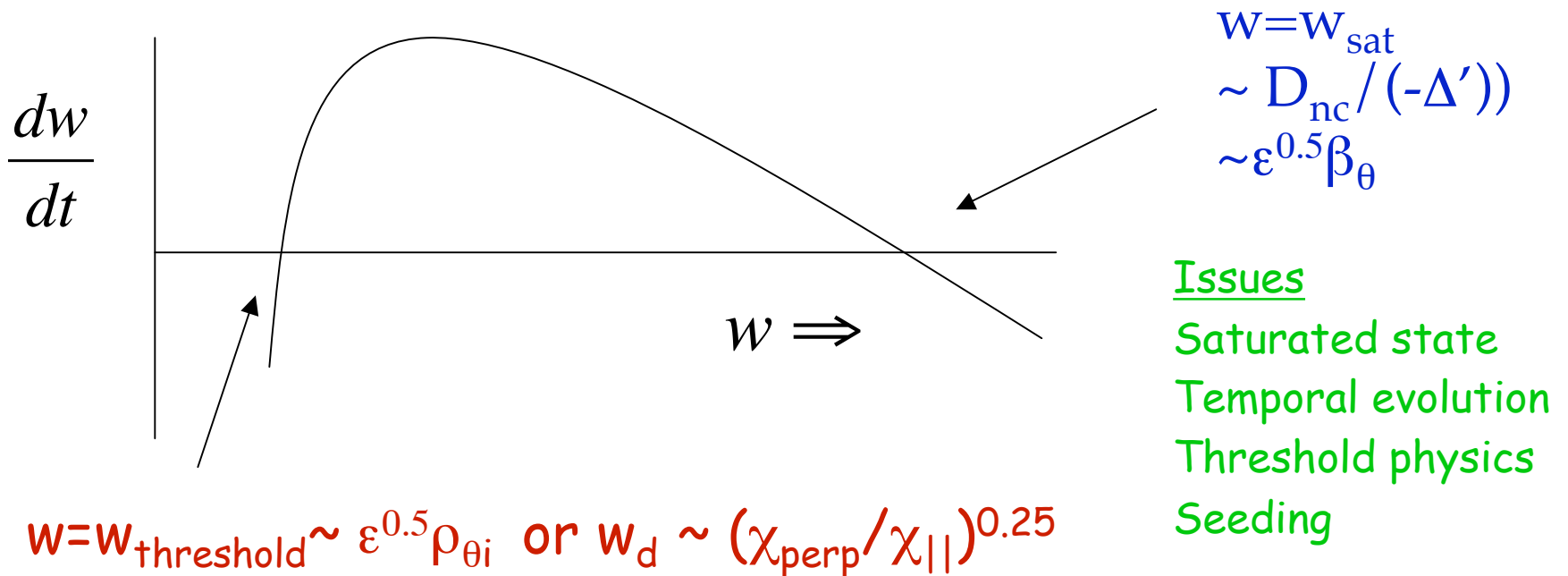
- Accurate predictions of “slow-MHD” island physics in high temperature plasmas requires the inclusion of a number of non ideal-MHD physical effects.
 - Neoclassical-like viscous forces
 - parallel viscous forces
 - gyro-viscosities
 - Anisotropic transport in helical geometry
 - Self-consistent Rf-source effects - ECCD stabilization

Outline

- NTM basics
- Neoclassical related closures
 - Bootstrap current drive - electron viscosity
 - Threshold physics
 - Neoclassical polarization currents - ion viscosity, gyro-viscosity, natural island frequency
 - Anisotropic heat conduction - heat flux
- ECCD stabilization - RF closures

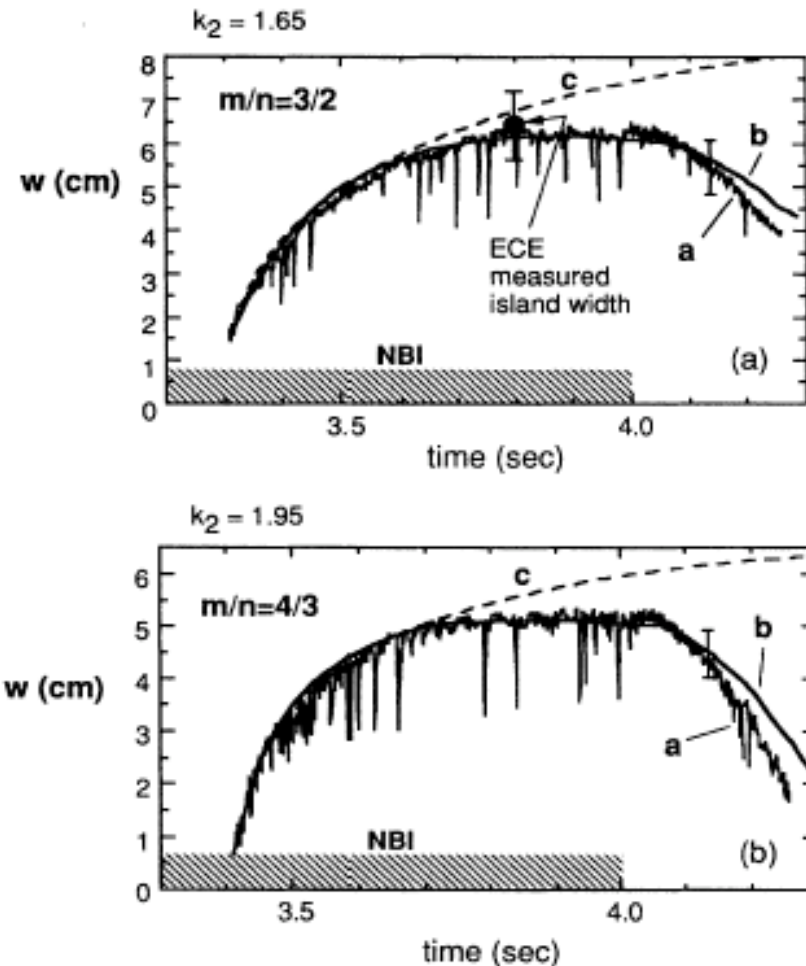
Elements of NTM physics

- Phase diagram illustrates different aspects of NTM evolution



NTM physics has been binned into different parts of the problem problems

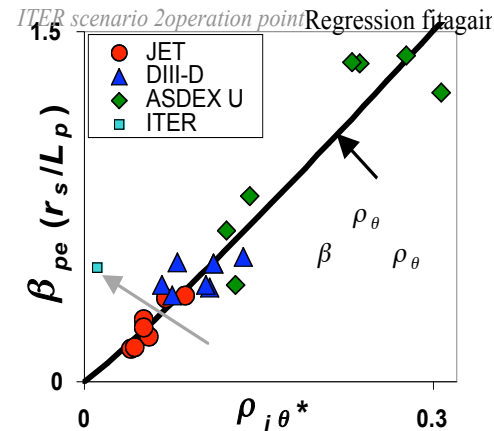
- Some elements of theory are in relatively good shape
 - Saturated state
 $w_{\text{sat}} \sim \varepsilon^{0.5} \beta_{\theta} \sim \text{local bootstrap current density}$ ---> in good agreement with experiment
 - Timescale of island evolution \sim resistive diffusion time through the island region ---> in good agreement w/expt
 - Observed on TFTR, JET, DIII-D, JT60U, AUG, etc



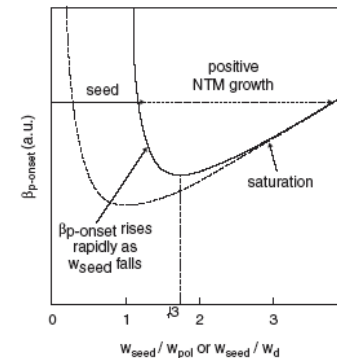
Chang, et al, PRL '95

Other elements of NTM physics are less well understood

- Threshold physics - NTM excitation requires $w > w_{\text{threh}}$
 - Empirically, critical β for NTM onset scales with ρ^*
 - > very bad for ITER
 - somewhat consistent with theory, but work remains
- Seeding processes
 - How to initiate $w > w_{\text{thresh}}$?
 - Some ideas, no single explanation for all observations - transient evolution of Δ' , electromagnetic coupling to outside MHD, transport changes, sawteeth period?



Buttery, et al, IAEA '04



Modified Rutherford equation provides a useful model to interpret NTM dynamics

- Nonlinear tearing mode dynamics initially worked out for resistive MHD theory - Rutherford '73

- Slowly evolving helical MHD "equilibrium"

$$\vec{J} \times \vec{B} = \nabla p, \quad \vec{B} \cdot \nabla \frac{J_{\parallel}}{B} = -\nabla \cdot \frac{\vec{B} \times \nabla p}{B^2}$$

Ψ^* = helical magnetic surface label

$$\Rightarrow \frac{J_{\parallel}}{B} = \frac{\langle \vec{J} \cdot \vec{B} \rangle_{\Psi^*}}{\langle B^2 \rangle_{\Psi^*}} + \text{helical Pfirsch - Schluter currents}$$

Local helical equilibrium defined by two flux functions - $p(\Psi^*), J(\Psi^*)$

- Resistive Ohm's law

$$\langle \vec{E} \cdot \vec{B} \rangle_{\Psi^*} = \eta \langle \vec{J} \cdot \vec{B} \rangle_{\Psi^*} \Rightarrow -\langle \frac{\partial \tilde{A}}{\partial t} \cdot \vec{B} \rangle_{\Psi^*} = \eta \langle \vec{J} \cdot \vec{B} \rangle_{\Psi^*}$$

- Matched asymptotics - Rutherford equation - with Pfirsch-Schluter effects (Kotschenreuther et al '86; CCH '99)

$$k_1 \frac{dw}{dt} = \frac{\eta}{\mu_0} \left[\Delta' + \frac{D_R^*}{w} \right]$$

$$D_R^* = (E + F + H^2) / (\alpha_s - H)$$

Neoclassical electron viscosity brings in bootstrap current effects

- Bootstrap currents - viscous damping of poloidal electron diamagnetic flow
 - Toroidal equilibrium - relevant neoclassical closures are the flux-surface average of parallel viscous forces

$$\begin{pmatrix} \langle \vec{B} \cdot \nabla \cdot \vec{\pi}_{\parallel} \rangle \\ \langle \vec{B} \cdot \nabla \cdot \vec{\Theta}_{\parallel} \rangle \end{pmatrix} = mn \langle B^2 \rangle \begin{pmatrix} \mu_{00} & \mu_{01} \\ \mu_{10} & \mu_{11} \end{pmatrix} \begin{pmatrix} U_{\theta} \\ q_{\theta} \end{pmatrix}, \quad \mu \cong \sqrt{\epsilon} \nu, \quad U_{\theta} = \frac{v_{\parallel}}{B} + \frac{\vec{v}_{\perp} \cdot \nabla \theta}{\vec{B} \cdot \theta}$$

- For electrons, $U_{qe} = U_{qi} - J_q / ne \cong - J_{\parallel} / (Bne) - F(dp/d\psi) / (neB^2)$
Alters electron force balance - Ohm's law

- For islands, the helical current is produced from the helical deformation of plasma profiles (Carrera, et al '85, ...

$$\langle \vec{B} \cdot \nabla \cdot \vec{\pi}_{\parallel e} \rangle_{\Psi^*} \cong \langle \vec{B} \cdot \nabla \cdot \vec{\pi}_{\parallel e} \rangle_{axi} \frac{1}{p_o} \frac{dp}{d\Psi^*} \langle \partial_x \Psi^* \rangle_{\Psi^*} + (\dots) J_{\parallel}$$

$$\cong \mu_e \frac{m_e}{e} F \langle B^2 \rangle \frac{dp}{d\Psi^*} \langle \partial_x \Psi^* \rangle_{\Psi^*} + (\dots) J_{\parallel}$$

Inclusion of neoclassical viscosity leads to neoclassical tearing mode instabilities

- Modified Ohm's law

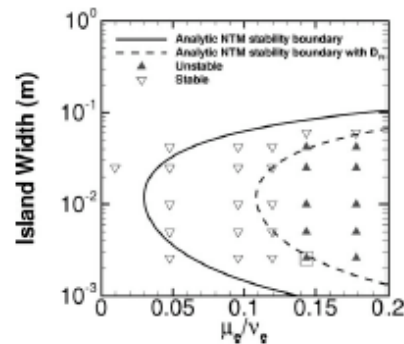
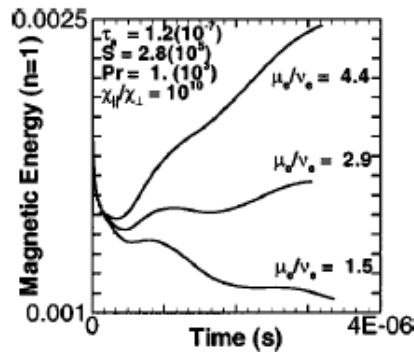
$$\langle \vec{E} \cdot \vec{B} \rangle_{\Psi^*} = \eta \langle \vec{J} \cdot \vec{B} \rangle_{\Psi^*} - \frac{1}{ne} \langle \vec{B} \cdot \nabla \cdot \vec{\pi}_{\parallel e} \rangle_{\Psi^*}$$

- Modified Rutherford equation (Carrera et al, '85, Qu and Callen '86)

$$k_1 \frac{dw}{dt} = \frac{\eta}{\mu_o} \left[\Delta' + \frac{D_{nc} + D_R^*}{w} \right] \quad D_{nc} > 0 \text{ for } dq/dr > 0, \text{ generally } D_{nc} > |D_R^*|$$

- Heuristic model suggested to facilitate NIMROD simulations of NTMs (Gianakon, et al '02)

$$\nabla \cdot \vec{\pi}_{\parallel e} = \mu_e m_e n_e \langle B^2 \rangle_{axi} \frac{\vec{v}_e \cdot \vec{e}_\theta}{(\vec{B} \cdot \vec{e}_\theta)^2} \vec{e}_\theta \quad \vec{e}_\theta = \sqrt{g} \nabla \xi \times \nabla \psi = \frac{\nabla \xi \times \nabla \psi}{\vec{B} \cdot \nabla \theta}$$



The heuristic model has a number of positive features

- Properties of heuristic model

- Manifestly dissipative, as it should be
- Yields correct perturbed bootstrap current for sufficiently small islands
- No toroidal flow damping; damps poloidal flows

$$\nabla \zeta \cdot \nabla \cdot \vec{\pi} = 0$$

- Gives neoclassical particle transport

$$\Gamma \cdot \nabla \psi = \frac{\vec{B} \times \nabla \cdot \pi_{\parallel e}}{B^2} \cdot \nabla \psi \neq 0$$

- Relies on coordinate system of the initial axisymmetric equilibrium - with larger island - deformed equilibrium? - neoclassical toroidal viscosity? (Shaing '02)

Polarization currents alter island stability

- In resistive MHD, rotating islands add polarization effects which alters island evolution

$$\vec{B} \cdot \nabla \frac{J_{\parallel}}{B} = -\nabla \cdot \frac{\rho \vec{B}}{B^2} \times \frac{d\vec{v}}{dt}$$

- In sheared slabs with cold ions, modified Rutherford equation ($w > \rho_L$) - Smolyakov '89

$$k_1 \frac{dw}{dt} = \frac{\eta}{\mu_o} \left[\Delta' + \frac{d_{pol}}{w^3} \right] \quad d_{pol} \sim \rho_i^2 \beta_{\theta}$$

- A number of folks are looking at various 2-fluid, cold ion, sheared slab modifications of this (Waelbroeck, et al '03; Ottaviani et al '05, etc.)

Neoclassical physics "enhance" polarization currents

- Neoclassical physics dominate the MHD polarization currents

$$\vec{B} \cdot \nabla \frac{J_{\parallel}}{B} = -\nabla \cdot \frac{\vec{B} \times \nabla \cdot \vec{\pi}_i}{B^2} \cong \frac{\partial}{\partial \psi} \frac{F}{B^2} \vec{B} \cdot \nabla \cdot \pi_{\parallel i}, \quad \vec{B} \cdot \nabla \cdot \pi_{\parallel i} \cong -m_i n_i \vec{B} \cdot \frac{d\vec{v}}{dt} \cong m_i n_i F \frac{d}{dt} \left(\frac{\partial \phi}{\partial \psi} + \frac{1}{ne} \frac{\partial p_i}{\partial \psi} \right)$$

$$\Rightarrow \nabla \cdot \frac{\vec{B} \times \nabla \cdot \vec{\pi}_i}{B^2} \cong \frac{B_{\zeta}^2}{B_{\theta}^2} \nabla \cdot \frac{\rho \vec{B}}{B^2} \times \frac{d\vec{v}_{\perp}}{dt}$$

- The q^2/ε^2 enhancement of the dielectric tensor - neoclassical MHD (Callen, et al '86)
- For lower collisionality regimes $\nu < \omega/\varepsilon$, the enhancement is down by $\varepsilon^{3/2}$ (Wilson, et al '96)

$$\nabla \cdot \frac{\vec{B} \times \nabla \cdot \vec{\pi}_i}{B^2} \cong \frac{q^2}{\sqrt{\varepsilon}} \nabla \cdot \frac{\rho \vec{B}}{B^2} \times \frac{d\vec{v}}{dt}$$

Physics - at low collisionality, infrequent collisions across trapped/passing boundary eliminates the passing particle enhancement of the banana's "diamagnetic" current

Strength of neoclassical enhancement depends on collisionality

- Modified Rutherford equation with neoclassical polarization currents

$$k_1 \frac{dw}{dt} = \frac{\eta}{\mu_o} \left[\Delta' + \frac{D_{nc} + D_R^*}{w} + \frac{D_{pol}}{w^3} g\left(\epsilon, \frac{v_i}{\epsilon\omega}\right) F(w) \right]$$

- Valid for $w > \epsilon^{0.5} \rho_{\theta i}$

$$D_{pol} \cong \epsilon \rho_{\theta i}^2 \beta_{\theta} \left(\frac{d \ln p}{d \ln q} \right)^2, \quad g\left(\epsilon, \frac{v_i}{\epsilon\omega}\right) \begin{matrix} \rightarrow 1 & v_i \gg \epsilon\omega \\ \rightarrow \epsilon^{1.5} & v_i \ll \epsilon\omega \end{matrix}$$

- For $F(w) < 0$, neoclassical polarization currents are stabilizing and provides a threshold island $w_{hresh} \sim \epsilon^{0.5} \rho_{\theta i}$ --
-- crudely consistent with $\beta_{onset} \sim \rho_i^*$

- The "collisional" version of this model should be described by the heuristic closure scheme

$$\nabla \cdot \vec{\pi}_{\parallel i} = \mu_i m_i n_i \langle B^2 \rangle_{axi} \frac{\vec{v}_i \cdot \vec{e}_{\theta}}{(\vec{B} \cdot \vec{e}_{\theta})^2} \vec{e}_{\theta}$$

Stability properties of polarization effects depend on frequency: gyroviscosity is important

- Two-fluid effects are important in evaluating the neoclassical polarization effect

$$D_{pol} \sim \omega(\omega - \omega_i^*)$$

- Origin of this dependence is two-fluid ion flows + gyroviscous cancellation

$$\vec{v}_{i\perp} = \frac{\vec{E} \times \vec{B}}{B^2} + \frac{\vec{B} \times \nabla p_i}{neB^2},$$

$$\frac{d}{dt} \vec{v} + \nabla \cdot \pi_\perp = \left(\frac{\partial}{\partial t} + \vec{v}_i \cdot \nabla \right) \vec{v} + \nabla \cdot \pi_\perp = \left(\frac{\partial}{\partial t} + \vec{v}_{E \times B} \cdot \nabla \right) \vec{v}_i$$

$$D_{pol} \sim \omega(\omega - \omega_i^*)$$

- Neoclassical polarization currents are stabilizing for $\omega(\omega - \omega_i^*) < 0$ --- threshold physics requires finite T_i and island rotation frequency in the range $0 < \omega < \omega_i^*$
 - First principal calculations of ware a topic of current reserach (Shaing '02; CCH '03; Waelbroeck and Fitzpatrick '05; CCH APS '05)

Anisotropic transport properties can also produce a threshold island

- NTM instability relies on self-consistent deformation of plasma profiles -
 - rapid equilibration along field lines relative to cross-field transport (Fitzpatrick, '95; Gorelenkov et al '96)

$$\frac{\partial p}{\partial t} = \dots \nabla \cdot [\hat{b}\hat{b}\chi_{\parallel} + \chi_{\perp}] \nabla p$$

- Characteristic length scale $w_d \sim (\chi_{\text{perp}}/\chi_{\parallel})^{0.25}$

$$k_1 \frac{dw}{dt} = \frac{\eta}{\mu_o} \left[\Delta' + \frac{D_{nc} w}{w^2 + w_d^2} + \frac{D_{pol}}{w^3} \frac{\omega(\omega - \omega_i^*)}{|\omega_i^*|^2} \right]$$

- Parallel heat flux in island-like geometry has non-local feature (Held et al, '03)

$$q_{\parallel} = \int dL' \kappa(L, L') T(L')$$

A number of open issues related to correct modeling of NTM seeding, dynamics, etc

- Local closures for viscosity, gyro-viscosity, heat flux in a slowly evolving helical 3-D equilibrium state
- Seeding physics
 - Other MHD events (sawteeth, ELM's, fishbone, etc,)
 - Prediction of island rotation frequency/ momentum evolution
 - Differential rotation of toroidally coupled magnetic perturbations
 - ?Field error/resistive wall physics
 - ?Self-consistent neoclassical toroidal viscosity due to evolving instability
 - Transport physics in the presence of islands
- Effect of islands
 - Transport in self-consistent equilibrium with islands/stochasticity
 - Effect on energetic particles
 - Self-interaction with rf

RF physics can be treated as a closure problem

- Inclusion of RF operator modifies kinetic theory

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \frac{q}{m} (\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f}{\partial \vec{v}} = C\{f\} + Q_{rf}\{f\},$$

- Moment equations are modified

$$mn \frac{d\vec{v}}{dt} = nq(\vec{E} + \vec{v} \times \vec{B}) - \nabla p - \nabla \cdot \vec{\pi} + \vec{R} + \vec{F}_{rf}, \quad \vec{F}_{rf} = \int d\vec{v} m\vec{v} Q_{rf}\{f\}$$

- Ohm's Law ($m_e \cong 0$)

$$\vec{E} + \vec{v} \times \vec{B} = \left(\frac{\vec{J} \times \vec{B} - \nabla p_e}{ne} \right) - \frac{1}{ne} \nabla \cdot \vec{\pi} + \eta \vec{J} + \vec{F}_{rf}$$

- Relevant quantity for modified Rutherford equation is the helical flux surface average of Ohm's law (CCH and JDC '97; Zohm '97; Sauter '04)

$$\langle \vec{E} \cdot \vec{B} \rangle_{\Psi^*} = \eta \langle \vec{J} \cdot \vec{B} \rangle_{\Psi^*} - \frac{1}{ne} \langle \vec{B} \cdot \nabla \cdot \vec{\pi}_{\parallel e} \rangle + \langle \vec{F}_{rf} \cdot \vec{B} \rangle_{\Psi^*}$$

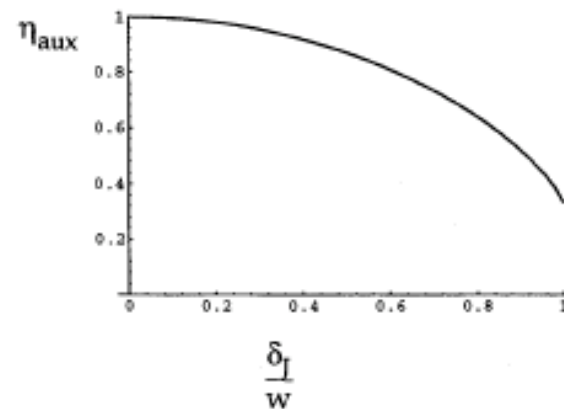
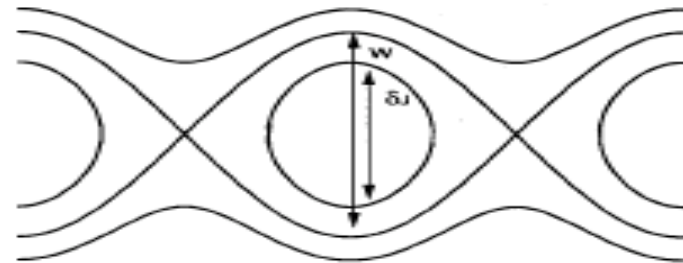
Phasing the ECCD improves the efficiency of eliminating NTMs

- ECCD currents equilibrate along helical flux surfaces

- Preference to localizing current inside separatrix at O-point

- Steady current drive works, but it is not as efficient ~ 3 in improvement

- Phased current current has only been tentatively attempted on present-day tokamaks - a high priority set of experiments on DIII-D, AUG



Theoretical modeling includes an ECCD contribution to modified Rutherford equation

- Modified Rutherford equation with rf-current drive

$$k_1 \frac{dw}{dt} = \frac{\eta}{\mu_o} \left[\Delta' + \frac{D_{nc} w}{w^2 + w_d^2} + \frac{D_{pol}}{w^3} \frac{\omega(\omega - \omega_i^*)}{|\omega_i^*|^2} - \Delta_{cd} \left(\frac{w}{\delta_{CD}} \right) \right]$$

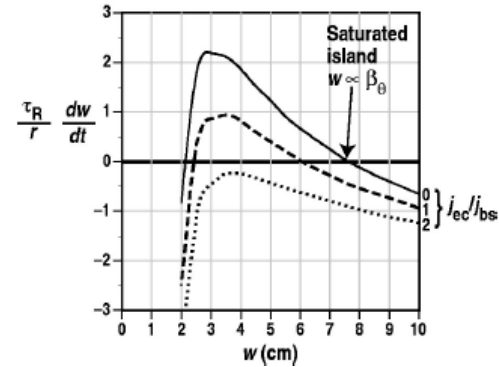
$$r_s \Delta_{cd} = -\frac{16\mu_o L_q I_{cd}}{\pi B_o \delta_{cd}^2} \eta \left(\frac{w}{\delta_{cd}} \right)$$

$$\eta \left(\frac{w}{\delta_{cd}} \right) = \frac{\delta_{cd}^2}{w^2} \frac{\int_{-1}^1 d\psi J(\psi) W(\psi)}{\int_{-1}^1 d\psi J(\psi) V(\psi)}$$

$$J(\psi) = \frac{1}{V(\psi)} \int d\alpha \frac{J_{cd}(\psi, \alpha)}{\sqrt{\psi + \cos^2(m\alpha/2)}}$$

$$W(\psi) = \int d\alpha \frac{\cos(m\alpha)}{\sqrt{\psi + \cos^2(m\alpha/2)}}$$

$$V(\psi) = \int d\alpha \frac{1}{\sqrt{\psi + \cos^2(m\alpha/2)}}$$



- ECCD stabilization has been successful in stabilizing NTMs (LaHaye '02)

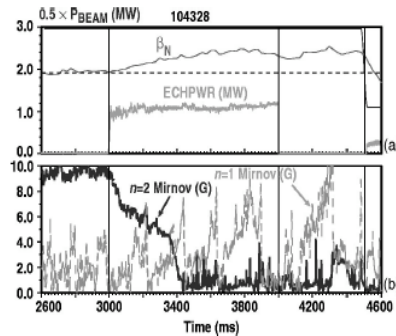


FIG. 5. Complete suppression of an $m/n=3/2$ NTM by ECCD in the presence of continued uncoupled sawteeth (2 gyrotrons, $P_{rf}/P_{beam} \approx 1$ MW/t MW, fixed B_T and R_{loop}). (a) The β_N and ECH power, (b) $n=2$ (solid) and $n=1$ (dashed) integrated Mirnov amplitudes.

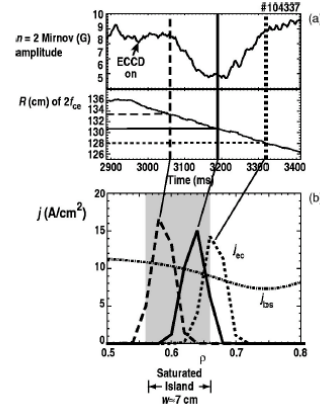


FIG. 4. (a) A B_T sweep to move the $2f_{ce}$ location past the island. ECCD on at 3000 ms (2 gyrotrons for ≈ 1 MW injected), (b) calculated j_{ec} from TORAY-GA [at the three different times indicated in (a)] and j_{bs} from ORBITO code reconstruction [at the middle time indicated in (a)]. The island width and location from the ECE radiometer at the first time indicated in (a).

There are a number of modeling issues associated with rf-modeling

- Logic from a MHD perspective - turn the rf-term as a closure problem. A couple of issues/opportunities arise:
 - With lots of rf/beams, etc - non-Maxwellian distribution - viability of expansion - Laguerre energy Polynomials?
 - RF modeling is generally 2D - how do we account for 3D with island induced $E_{||}$ and helical magnetic fluctuations?
 - Self-consistent evolution on transport timescale with particle, momentum, energy inputs?
 - ECCD efficiency requires an accurate temporal evolution of electron slowing-down etc. Particularly an issue for phased current drive (Giruzzi, et al) - phased ECCD is crucial for efficient ECCD stabilization on ITER.
 - Numerical instabilities from energetic particle components that significantly modify Ohm's law (Jardin and Ignat '95)

Summary

- Heuristic model for parallel viscosity has a number of desirable features
 - Correctly reproduces perturbed bootstrap current and polarization currents
 - Manifestly dissipative, damps poloidal flows, no toroidal flow damping, gives neoclassical cross-field particle transport
- To properly describe the neoclassical polarization current effect, it requires
 - Neoclassical ion poloidal flow damping
 - Two-fluid physics - Hall-MHD terms in Ohm's law
 - Gyroviscosity - gyroviscous cancellation to yield proper frequency dependence
- The inclusion of RF-forces on the electron momentum balance equation can be treated as a closure problem, but a number of issues need to be resolved